Voices of Teachers and Teacher Educators

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About the Journal

The journal 'Voices of Teachers and Teacher Educators', an initiative of the Ministry of Human Resource Development (MHRD), is now being co-ordinated by the NCERT. The Journal highlights the vital role of teacher education in India, as the country is poised to provide quality education to all its children, irrespective of gender, caste, creed, religion and geography. The National Curriculum Framework (NCF)-2005, the National Curriculum Framework for Teacher Eduation (NCFTE)-2009 and the Right of Children to Free and Compulsory Education Act (RTE)-2009 all reflect this commitment and underline the principles that make such an effort necessary and also spell out the strategies for it. The challenge is to augment the role of teachers in shaping the social transformation that India is witnessing, have a long lasting impact on the quality of education, and making education equitable. Teachers and all those concerned with education need to recognize that their ownership and voices are important and that they can and do learn not only from their own experiences but also from each other through collective reflection and analysis. The Journal attempts to lend voice to teachers, teacher educators, researchers, administrators and policy makers in varied institutions such as schools, Cluster Resource Centres (CRCs), Block Resource Centres (BRCs), District Institutes of Education and Training (DIETs), Institutes of Advanced Studies in Education (IASEs), Colleges of Teacher Education (CTEs), State Councils of Educational Research and Training (SCERTs), etc., and make their engagement visible in accomplishing extraordinarily complex and diverse tasks that they are expected to perform. Contributions to the Journal are welcome both in English and Hindi. Voices is an e-Journal and we hope to circulate it widely. We also look forward to suggestions and comments on the articles published. The views expressed and the information given are that of the authors and may not reflect the views of the NCERT.

Call for Contributions

This biannual publication is for all of us: teachers, teacher educators, administrators, researchers and policy makers. It is to provide a platform and also to build a network for our voices, ideas and reflections. Since the idea is to make this journal reflect all our voices, it would fulfill its purpose if we contribute to it in as many ways as we can. We look forward to all of you contributing with your experiences, questions, suggestions, perspectives as well as critical comments on different aspects of teacher education and schooling. Your contributions could be in the form of articles, reports, documents, pictures, cartoons or any other forms of presentation that can be printed. This could also be through comments and reflections on the current issue for improvements of the publication to make this a participative endeavour and improve its quality. We look forward to your inputs to make this journal truly reflective of our voices. We look forward to receive your contributions for the next issue by 30th April, 2017. We also look forward to comments and suggestions. The next issue would be focused on Curriculum and its practice. The contributions can be sent to the following:

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Editorial

The Voices of Teachers and Teacher Educatos (VTEE) is back after a long time with a spirit that has freshness and carries the rich fragrance of its heritage and tradition. It was delayed due to unforeseen circumstances. This issue is a special issue on teaching-learning of Mathematics. The tradition of mathematics in India goes back to the beginning of the formulation of the discipline. It has risen and ebbed and the journey is marked by periods of vigorous activity and new directions. In recent years, the name of Srinivasa Ramanujam is enough to make all heads bow in reverence and respect to the mind and the spirit that he carried for mathematics. This issue is dedicated to him even though it does not carry any article on him or about him. Almost all the articles in this issue are on mathematics teaching-learning. It also carries two contributions that are not directly linked to mathematics but have been with the editors for a long time and required to be shared.

The VTTE is fortunate to have teachers, teacher educators, and researchers contributing and sharing their analysis and discussions of their experiences. Even a break has not dented the enthusiasm and support of the readers and the contributors.

The richness and the diversities of ideas that, the issue shares, indicate the commitment and the versatility that exists on ground. While each of the contributions is valuable in itself, it also reflects diversities in the perspectives and reaffirms the principles focussed on in the National Curriculum Framework (NCF)-2005 as well as Position Paper of the National Focus Group on Mathematics Teaching. The journal will give priority to the Voices of those teachers who rarely find opportunities to report their work. Any reflection on the school and the classroom carries the perspective and the lens of the person and has the stamp of his/her personality.

India is characterized by diversities. However, its multiple realities have an underlying common thread that is to be identified and respected. Often these diversities and their genesis are in conflict with the 'convenient' formulations of truth that gain ground realities. Any effort to appreciate the nuances would have to critique the 'perceptions' as well as the 'opinions'. Mathematics, inspite of its deep embeddedness and roots, remains a revered and feared area. The fear is manifested not just in Indian classrooms, but in most classrooms of the world. This issue of Voices carries experiences, explorations and views on mathematics and its teaching and learning from a variety of practitioners.

The first article by R. Ramanujam reflects on the trajectory of transition from the concrete and the direct experiences to the abstract ideas. He walks us through another experience of generalising and abstracting and in a subtle way demonstrates the universal ability in humans to build abstractions of mathematics. The article shows that how making these transitions is helped by the facilitator who remains aware and shows positive attitude towards the learners and their environment. He points out that in learning mathematics there are many such transitions and the learners must be helped in this process. It is those learners who fail to make the transition get left behind. There is thus a need to articulate and share these transitional stages so that learners can be scaffolded. The article has hidden in it many ideas that are at the core of the position paper on Teaching of mathematics mentioned earlier.

The issue carries contributions dealing with what mathematics teaching is about and the materials that would enable learning more effectively. Dewan points out that the material and/or methods have to be directed by the purpose. Emphasising that mathematics by nature is abstract and deals with abstract entities, relations between them and operations on the entities assume significance. He further argues that the materials and concrete models of experiences are temporary and the learner must throw away these crutches and deal with the abstractions. He points out that the concrete and the abstract are spiral in nature. What is abstract for the early primary becomes concrete for the next as the learners transit to the higher stages of abstraction. He underlines the use of materials with care and avoidance of over-dependence on them for both the teachers and the learners. In another article, Subramanium points out that learning of mathematics includes making sense of its use in asking and answering questions in real situations. He emphasizes on meaning making, starting with experienced situations, allowing students to explore their own ways of solving problems and only then helping them access the powerful generalised ideas and methods. These two papers and the paper by Ramanujam indicate the need to recognise that there are transitions that have to be made by the learners for going to generalised numbers and then from arithmetic to algebra and so on.

In her article, Haneet focuses on the critical ability needed by the facilitator to make transitions possible. She argues that without conceptual understanding and an ability to mathematise, teachers can not help the learners. She suggests the need to engage teachers in the act of thinking mathematically. She further argues for constructively challenging teachers' existing mathematical cognition through tasks that require thinking, reasoning and making conceptual connections.

In a similar vein Utpal suggests that the non-recognition of this critical ability and the tendency to force memorisation and short cuts place stress and burden on children. This results into children not able to learn mathematics in subsequent years. His article, therefore, points out the danger of spoon-feeding leaners and helping them somehow tide over the present assessment. He also argues that this can not change unless the teachers who teach children at the primary level themselves have a better understanding of mathematics. The paper by Goswami focuses on children's errors to understand their current knowledge. She argues that if the teachers analyse the errors of the students, it helps them assess the learning and the strategies they are using. This would help them choose a more appropriate strategy. This analysis is possible easily provided the teacher makes the effort and is familiar with the concepts and their path of development.

The spectrum of this issue also includes the contributions from two teachers, Shehnaaz and Mukesh, who have spoken about their attempts to do mathematics differently in their classrooms. The exploration of Mukesh shows how teacher can be excited about mathematics and learn from the questions asked by the students. Shehnaaz points out the importance of textbooks, what they reflect and the effect they have on the teacher. Good materials can excite and challenge the teacher and the other just makes him/her follow procedures and get bored.

The classroom experience of Aaloka also shows the joy that can imbue a mathematics classroom. A sensible use of materials and an open exploratory pedagogical classroom helps building the climate and learning of mathematics considerably. Materials are not the ends but the means and an attitude of curiosity, exploration with an understanding about mathematics in the teacher/facilitator

can make learning possible and reduce the fear of mathematics. Yashwendra brings out the complexity of teaching situations in a school where children are not regular. The gradual building of dialogue by allowing children to think and formulate ideas makes not only the mathematics classroom interesting but also shows that children can deal with abstract and do not have to be flummoxed by letter numbers.

Vijayan reports a survey about the teachers' understanding of what is done in the classroom and what they think they need by way of capacity building. This is in the light of the NCF 2005 and the Pedagogy –Content-Knowledge (PCK). The paper by Sanjay Gulati is on the use of ICT in Mathematics. Sanjay argues for use of flexible and free ICT tools like Geo-Gebra that are available in many languages and leads us through some steps as to how it can be done. The last paper, titled 'From Kothari Commission to Contemporary System of School Education', analyses and critiques the role of parents in education and links it to the present day education policies. A book review of Toto Chaan is also reported in the journal.

We also share information about recent initiatives on Mathematics Education as the focus of interactions among students, teachers and teacher educators. This includes the National Conference on Mathematics Education organised by RIE, Bhubhneshwar and about the Ganit Saptah that is to be held by all RIEs and other institutions. The third is a report about voluntary forums of teachers interacting around mathematics.

Editorial Committee

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Transitions in School Mathematics

The movement from 'real world' problems to formalization to mathematical *Discovery includes many transitions.* (V.I. Arnold). `

An Exploration

What follows is an account of an adult education class, I was involved in, during 1991 in one of the slums of what was then Madras city. The tenements did not have piped water, there were pumps every 100 metres (or so) from which water was collected. In addition, tankers would arrive periodically bringing water, and residents stored them in buckets (and containers called *kudam*s).

Many learners were often late to class, and the standard reason was the water truck. Someone pointed out that the same story seemed to be told irrespective of which day of the week it was. The class met on Mondays, Wednesdays and Fridays, shifting to Tuesdays, Thursdays and Saturdays some weeks (for a variety of reasons). And yet, the water truck as reason seemed to be uniform, though the trucks did not come every day. This observation led to a very interesting discussion. Water trucks came every third day at that time. Assuming that the trucks came on a Monday, learners realised that within three weeks they would have come on all days of the week.

Speculatively, we asked if the arrival would span all days of the week if the trucks came on alternate days. This was indeed verified to be true. A natural supplementary was to ask whether the observation held for trucks coming every fourth day. At this point, learners had difficulties, so we drew a diagram: the days of the week on a circle, and lines taking us from Monday to Friday to Tuesday, etc. The result was a single closed trajectory that visited all the day-vertices exactly once.

A logical next question was about trucks coming every fifth day (and then, to trucks coming every sixth day), but learners found the question ill motivated and most of them simply refused to "waste time" on these considerations. But then someone pointed out that we could still see what kind of *picture* obtained, whether it was similar to the closed curve we already had for 'every fourth day'. This suggestion met with an enthusiastic response, and the curves were drawn. The conclusion that a "full visit" cycle obtained for "whatever" frequency of truck visit seemed heartening to the learners. I tried to spoil the party with the suggestion that trucks arriving every seventh day would always arrive on one single day and thus the statement was true only for frequency varying from 1 to 6. But this was indignantly dismissed as "obvious and meaningless", since only frequencies from 2 to 6 were "interesting".

The next exercise was to consider a different arrival event, but once every three hours on the clock. My clumsy

attempt at story-making met with derision and one of the learners said it was only about drawing pictures, so there was no need for stories! This led to a flurry of drawings, notebooks soon filled with circles and linear trajectories visiting vertices on them. The fact that a frequency of 5 led to a full visit on 12 vertices, but that frequencies of 2, 3, 4 led only to partial visits led to the conjecture that this was about division: if the frequency divided the total, only a partial visit would obtain, but if it did not divide, a full visit was sure. This conjecture was confirmed by 6 and 7 (hailed and celebrated at high decibel levels) but alas, falsified by 8 and 9. Most learners simply gave up and went home at this point.

But then a few persisted, and in a few days' time, not only did we have a rather large collection of drawings (some of them very beautiful), but we also had one of the bright learners identifying the pattern: full visits obtain exactly when the two numbers (frequency and total) were relatively prime (though not stated in this language). I tried to formalize the statement as a theorem, for any k and n, but most learners saw no point in that, seeing it as some mumbo jumbo. The few who were indeed curious that the statement would be true for any k and n, could not see how I could be sure for say 1500 points on a circle, with the curve visiting every 137th successor. I did try to explain that this was possible and that in some sense, that this was what Mathematics was all about, but I did not succeed in the effort.

Within a few years, this activity led to an interesting game with children. Seat n children in a circle, each child numbered 1 to n, remembering her number. A book is passed around, starting with the first child, passing to the k'th neighbour. This is supposed to go on until every child gets the book. Soon children realise that for some values of k and n, everyone gets the book, for some values they don't. Many conjectures are made, and invariably the pattern is discovered. Many pictures are drawn. I have now conducted this activity with many groups of children and teachers, and invariably the moment of discovery comes after these well-defined stages.

However, one thing is clear. In all these discussions, there definitely was argumentation and inference, though it never graduated to proof and universally quantified statements. On the other hand, limited to experimental verification in the small, the learners, be they adults or children, could play around with notions like curves, closed curves and orbits, without ever learning such vocabulary.

Transitions

We often speak of the need to go from the concrete to the abstract in elementary education, especially in the context of Mathematics. But often missed is the realisation that this is a deliberate transition, one that is neither natural nor obvious. A concrete situation or object can be abstracted in many ways, and in the class, we are picking up one particular abstraction (for perhaps very sound reasons). Moreover, after some repetition of such concrete instances abstracted, we want the child to deal with the abstraction per se, leaving behind the concrete realm altogether. This is what I am referring to as a *transition*, moving from one realm to another, often irreversibly.

For instance, when 20 rotis are to be divided equally among 5 persons, it makes sense to act out the division, giving one roti each until all the rotis are exhausted. But when faced with the problem 5624/703, it would be the wrong move to think of distributing 5624 items among 703 persons. Now, why on earth would anyone want to

solve such a division problem, at all ? It is highly unlikely that "real life" would ever present us with this problem. The need is entirely mathematical, that of dealing with abstractions like number, division and the patterns visible: 56 / 7 = 8 and 24 / 3 = 8 as well, so one can make a bold guess that the answer is 8, and verify it. Such a facility with abstractions is essential for Mathematics, and students who have not made the transition into this realm, who are yet in the concrete division realm, would find the Mathematics class slipping away from them.

Acknowledging and identifying these transitions is essential for Mathematics curriculum and pedagogy, both at the school and at the college level. Perhaps not surprisingly, these transitions are co-located with what are considered *difficult* topics for teaching/learning. Those who have made the transition need to be engaged in the new realm, those who are yet to make it be given more opportunities. There is no one unique way to make this transition either; recognizing that there are multiple pathways and renewed opportunities is important as well. Understanding these processes also offers hope for solutions to the difficulties mentioned above.

How does one recognize a transition in teaching/learning? Any concept or process that seems difficult to master but seems so obvious and easy once it has been mastered that it is hard to go back to the previous state of learning, involves a transition. This happens when we learn to swim or ride a bicycle. Once you acquire balance, it is almost impossible to return to the wobbly state. Once we learn to factor polynomials, or perform integration, it is impossible to return to the days of early algebra and work out things the way middle school teach us.

This observation lies at the heart of the disconnect between school Mathematics and university Mathematics. A central objective of Mathematics learning is to provide powerful tools that are amazingly general and reliable. When one is equipped with the tool and learns to use it, this renders previously used tools entirely irrelevant. School teaches trigonometry, without which trigonometric functions and calculus cannot be learnt. But having learnt calculus and linear algebra, there is never any need to return to almost any topic taught in school. Later when one learns to use compactness and continuity as a principle, it liberates one from some of the specifics in calculus. Thus the journey continues, and it is one of making many a transition comfortably.

A Map of Transitions

There are many points of transition in Mathematics education, all the way from the primary classroom to the undergraduate class at university. It will be presumptuous on my part to attempt any comprehensive list. Instead let me enumerate some glaring transitions and their pathways.

- Long division: Though multi-digit multiplication involves working out a procedure, it is sufficiently close to the corresponding concrete operation that a transition is not necessitated. Not so in the case of long division; the student deals with an abstract procedure whose correctness or justification becomes clear only after mastering the algorithm itself. But this is possible only if the student can handle the abstractions employed.
- Addition of fractions: When we add natural fractions, we can make stories around them, but when faced with a student who considers 2/3+ 3/5 to be 5/8, the need to make the transition is obvious. The problem

is not lack of understanding LCMs and common denominators, but about *fractions as entities* that we can perform operations like additions on.

- Arithmetic to algebra: This is perhaps the best acknowledged transition in the school curriculum, and algebra is introduced as generalized arithmetic. But even here there are many jumps not negotiated neatly, leading to fall and fracture. For instance, in *x+5=8,* the variable *x* is a single unique unknown number; in *x+y = 8* the variable *x* stands for many unknown numbers (though there are only 9 possibilities if *x* and *y* are positive integers); in *x+y = y+x,* the variable *x* could be any number whatsoever.
- *Additive to multiplicative reasoning*: While it is natural to consider multiplication as repeated addition in the primary school, this makes little sense when faced with *sqrt(2) X sqrt(3).* It is critical to begin seeing multiplication as scaling of some kind. Multiplicative reasoning is crucial to understand the growth of functions, for recognizing similarity in geometric objects and for recognizing and using transformations.
- *From the implicit infinite to the axiomatic infinite*: In school, the infinite is always around, but it is not confronted as such. College Mathematics begins with limits and continuity, by which time infinite objects and sequences are understood in terms of their properties. For instance, consider the question: why does *1/n* tend to zero as *n* becomes large ?
- *From working with a model to the abstract notion*: The decimal representation of real numbers is known to children in the high school. Unfortunately it gets forgotten that the representation is only a *model*, the notion itself is more general.
- *From the assumed infinite to the explicit finite*: In school, numbers are always around, as big as you want. When one is engaged in combinatorics or number theory problems, one has to work with the explicit finite, and this Is often considered difficuly.
- *Limits and continuity*: Perhaps the biggest experienced discontinuity for students is the epsilon - delta definition of continuity. This is in such an abstract realm, formulated for rigorous foundations, that the demand it makes in terms of a big leap causes many students to be left behind.
- *From inductive to deductive argument*: In middle school, the student is encouraged to observe patterns and generalize them to obtain formulas. Later on the formulas require derivations, proofs. This is due to a cognitive shift that has occurred, whereupon the student is subjected to a standard of proof that is more stringent than what was acceptable earlier.
- *Geometrical reasoning*: Euclidean geometry provides a wonderful opportunity to learn logic in school. The big difficulty in making the transition from factorizing polynomials to such deductions renders many students clueless, they don't know what to look for.
- *Probabilistic reasoning*: This is a unique departure from the rest of Mathematics that involve certainty (theorems). Abstraction and imagination well beyond observable phenomena require comfort with probabilistic reasoning.
- *Mathematical modelling*: This is a part of curricular, but the modelling

never challenges the student's mathematical conceptualization. Indeed, modelling may reintengrate several knowledge domains of Mathematics towards problem solving.

While these can be seen as problem areas that require our attention, there are many simple questions that can pose a big leap. For instance, it seems reasonable that dividing a ribbon of length *m* among three persons should get *m/3* units for each. But what about cutting it up into pieces *3 cm*long generating *m/3* pieces ? Is that possible?

Here are some more questions. Why is *0.99999....= 1* ? What are we assuming here? What is *pi^2,* really speaking ? (Try starting from *pi* as the ratio of circumference to radius of an arbitrary triangle and think of what it means to multiply it by another.)

Bigger than all this is the transition required in a student's predisposition, as she moves from problem solving as the way of obtaining answers to that for gaining insight or constructing arguments. That good problems are those whose solution lead to several new problems, is an essential aspect of doing Mathematics and students who achieve this understanding enter into a new way of thinking altogether.

All this has major implications for the teaching of Mathematics. In Felix Klein's words, Mathematics teachers suffer due to a *double discontinuity*. Many teachers had themselves not negotiated the transitions successfully and lack introspection on these difficulties. When they went from school to college, they moved away from school Mathematics never to return to it for conceptual need. But becoming a teachers requires a backward journey when most of the university Mathematics learnt seems irrelevant. We require knowledgeable teachers, but most teachers do not have personal experience of what it means to do Mathematics over time, exploring questions which have intellectual purpose, not only pedagogic purpose.

This also poses challenges for Mathematics curricula. Allotting equal space for all curricular units is like insisting that everyone should walk at the same speed everywhere. The terrain dictates our ease and speed, and so also is the terrain of mathematical learning: there are easy passes, little streams to jump over, but also brambles to cut through, rocks to climb, and pits to avoid. Once we have a good map on hand and prepare ourselves well, the trek is enjoyable and healthy.

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Building a Better Mathematics Classroom

Abstract

This Paper focuses on the holistic teaching-learning process for Mathematics as embedded in the current thinking on Mathematics education which argues for the child building her own understanding through the process of engagement with concepts in different ways including having opportunities to collectively construct definitions and examine and modify them, developing formulas or/and algorithms, proofs and solving new problems, building patterns and generalisation, creating and setting problems and puzzles, talking and expressing mathematical concepts in written form, etc.

1. Introduction

Whenever we think of the methodology of teaching a subject some basic aspects have to be kept in mind. These include:

A. The nature of the subject and the way it manifests itself and constructs and accepts new knowledge;

B. The feelings and attitude of the learners, teachers, the system and the community to the subject and how they view its importance and its learnability;

C. The understanding about how humans, particularly children, learn;

D. The background of children and teachers along with the kind of schools and the education system that exists;

E. The purpose for which we want to educate children, in this case in Mathematics;

F. The curriculum and the content of Mathematics for the classes children are in and the resources that can be used. The class-room depends on the way these are understood. It is not possible to discuss all of them in this but we would discuss the essential aspects of most of them. Each of these have been discussed briefly to lay down the contours and their implications on the class-rooms.

2. Nature of the Subject and its Manifestation

The nature of the subject and the way it manifests itself and constructs and accepts new knowledge informs the way classrooms must be constructed. Mathematics is considered different from other disciplines; less driven by context, more abstract and in comparison to other disciplines more hierarchical implying each idea is intricately linked to other mathematical ideas and objects. This would suggest a linear class-room content and process. However, the recent discourse on Mathematics education elaborates the above and this brings out nuances that qualify above statements. While these nuances are stated in many different ways, we choose to follow one that is also reflected in the position paper on Mathematics NCERT 2005. It points out:

- Mathematics learning is about knowing, finding and creating patterns, relationships and discovering other properties among mathematical objects.
- Mathematics is present in what we do and is best learnt through that.
- Knowing Mathematics means being able to create and use ideas to form new ideas for yourself. It is about leading learners to find solutions to

unfamiliar problems and to create new problems.

All these add to what Mathematics is and what is worth learning and teaching in it. These would be embedded in the chosen content areas like numbers, shapes, angles, data, functions, equations and their relationships to each other and to life, etc. This decade old formulation is far from being operational in the classrooms. The common notion of Mathematics continues to be limited to jugglery with numbers and different combinations of operations on them. The other areas are added as definitions and algorithms (standard and non-standard short-cuts). The attempt is to acquire enough bits and pieces to go through the exam without it all coming together to form a picture. In the absence of recognition of even the need to differentiate a mathematical statement from a non-mathematical statement or to understand what it means to generalise and to prove them. The students do not get a feel for the process of accepting mathematical knowledge as valid.

The other difficulty is that the NCF like statements are nuanced and has ideas about Mathematics that seem to suggest opposite ways. One argument that can be made is that Mathematics is fully a part of our life and has meaning only when completely embedded in that and hence as a utilitarian area of knowledge. There is a consequent over-focus on concrete materials and models. The above are however nuanced and have to be read together with other statements of the NCF. While Mathematics does relate to our lives and is always manifest in it, yet the form that it is seen in daily life has to be generalized and the underlying Mathematics abstracted from the experiences. It must be delinked from any particular context and from physical materials that may have been used as models for ideas. Validity of mathematical ideas is not established through multiple repeated observations to conclude the veracity of the statement. They have to be logically deduced from axioms and prior known results. Mathematics is about generalising rules and being able to see and understand them and its ideas emerge from precise presentation of the underlying concepts and their consistent use to build a hierarchically structured understanding.

3. Attitudes to Mathematics

"The feelings and attitude of the learners, teachers, the system and the community to the subject and how they view its importance and its learn ability also affects classrooms."_The present day perceptions of Mathematics as it is to be taught and learnt are characterized by some fundamental underlying commonalities:

- Mathematics is difficult and meant for a few intelligent children.
- Mathematics is not about understanding concepts but abilities to do.
- Mathematics is not useful to learn as it does not aid our day to day life except in the utilitarian sense of calculations and data.
- Mathematical knowledge and all other knowledge is learnt bit by bit through the process of simplification and explanation. These are what the child must reproduce exactly in the way they are given to her.
- Mathematics definitions, shortcuts and solutions to problems are to be explained and given to children and that they would then memorise and remember them.
- Repeated practice of the same algorithm on the same kind of problems can lead to conceptual understanding.

– In total therefore, Mathematics is a subject in which algorithms, facts and in fact even solutions to problems have to be remembered and reproduced.

These perceptions have led to a classroom methodology that believes that practice makes a learner perfect. An often quoted couplet about the constant rubbing of a rope against a stone for a long time resulting in a mark on the stone is used as an illustration of the maxim. We need to examine what this view means, especially with reference to our ideas and understanding of how children learn. It also leads to strategies that are stratified as most children considered not destined for Mathematics be only given short cuts to memorize and attention be given to those who are learning and hence also appear keener.

The above described commonalities run counter to not just the nature and purpose of Mathematics, but also principles of pedagogy, articulated in the NCFs, particularly in NCF 2005. NCF 2005's view, aligned with learnings from across the world, argues for conceptual and procedural understanding arrived through solving making new problems, alongwith formulation of own definitions and arguments. All this has ramifications for the role of the teacher, the teaching-learning process and the nature of resources .

The knowledge of children prior to school: A robust system to help all students learn Mathematics is helped by recognition of the extent to which Mathematics penetrates our lives. Any approximate or exact task of quantification, requires enumeration, estimation, comparison, scaling, use of the operations, conceptual ability to deal with space and spatial relations including transformations, visualising, mapping and projecting and so on. It is not only adults who learn and use all these but also children.

These are not however, learnt or even used in the formal way as in school and later Mathematics. Mathematics learnt and used in everyday life is often not useable in all situations and contexts that formal Mathematics considers similar. For example a newspaper vendor can track the papers sold and money collected; a vegetable seller or a paan seller can do all accounts needed for maintaining the shop, but would perhaps find it difficult to do the same type of mathematical processes when placed in another shop, even with the price list.

It is not that the Mathematics learnt in daily life is not at all generalised and abstracted. The situations presented are varied. The forms of generalisation, the way of reaching answers, the way of communicating, etc are all however, different from formal Mathematics. The important point is that some Mathematics is known to all children and each child not only acquires that, but develops her own strategies to deal with the Mathematics that she needs.

In this sense Mathematics has an important role in our life and our experience of it helps us learn more of it too. This also tells us that human children have the ability to learn to generalise, abstract, visualise and deal with quantities.

4. Learning Formal Mathematics

The above discussion suggests that Mathematics when acquired in a generalised and somewhat formal way would develop abstract thinking, logical reasoning and imagination. It would enrich life and provide new dimensions to thinking. The struggle to learn abstract principles would develop the power to formulate and understand arguments and the

capacity to see interrelations among concepts. This enriched understanding would also help us deal with abstract ideas in other subjects and in our lives. Supporting us to understand and make better patterns, maps, appreciate area, volume and similarities between shapes and implications of their sizes. As Mathematics includes many aspects of our life and our environment, the symbiosis, inter dependence and inter-relationship between learning and using Mathematics in life needs to be emphasised.

We also need to ensure that even though Mathematics deals with a lot of symbols, abstractions, logic, spatial perceptions, generalisations, patterns and rules, it must not appear as difficult and meaningless to children. A classroom must give children a feeling that they are doing something meaningful that relates to reality around particularly at the primary school stage without getting trapped in utilitarianism. We must embed Mathematics and problems in it, for children to do through meaningful enjoyable situations and space for childrens' creativity and allow for multiple strategies thereby valuing children's articulation and logical formulations, even though not fully aligned. The classroom process must allow and demand that children create tasks, questions and problems for the classroom discussion. The NCF 2005 would suggest that the way forward is towards Mathematics class-rooms where children have opportunities to explore mathematical ideas and models not necessarily nor primarily concrete.

The purpose of Mathematics learning thus becomes developing the ability to explore mathematical entities and add to what is known. A growing ability extending beyond the classroom to help the learner mathematise her experiences. Using concepts to perceive the world differently and widening the manner of organising and analysing experiences. Helping learners find mathematical ideas and find instances of where mathematical ideas may be useful and what they can tell us. We would like them to be able to create new ways to solve problems and develop the ability to find solutions to new problems. In order to be able to recognise situations where mathematical ideas may be useful, we must be able to convert them into mathematical descriptions and present them as mathematical expressions or statements that can be solved and interpreted.

The background of children and teachers, kind of schools and the education system that exist for learning, and choice of materials and tasks: While the principle of universal education and equity in the nature and quality of education demands that all learn Mathematics and the evidence of engaging with Mathematics in daily life shows that all can learn it.

The current common belief and attitude about is that Mathematics can be understood only by those few who are bright and 'intellectually' well endowed. Only they can understand and for the rest therefore it has to be memorisation of solutions, or short cuts or algorithms and formulas. NCF 2005 recognises the abstract nature of mathematical ideas and the distance some children may have from some of these, but considers these as constraints to be overcome through concern and respect for the child, her knowledge, language and culture. In all pre-NCF 2005 documents (except perhaps the NCF 2000, where it was not so clear) one common feature was the impression that Mathematics is not easy to learn and that many children will fail to learn it. The kind of ideas about the resources and the methods to help children learn were also different.

5. The curriculum and the content of Mathematics: While there is an agreement that Mathematics must be learnt by all, but what to teach in Mathematics remains contentious. The emphasis in Nai Talim and in the Kothari Commission was different as being core in one and useful for Science and engineering in the other. National policy on education 1986 focussed on this and ability to use Mathematics in daily life. The NCF 2000 underlined the importance of the utilitarian purposes for life of Mathematics. With the scope largely around numbers and their use in market, in mensuration or other areas in life.

The NCF 2005 with an emphasis on abstraction, use of logical forms, grasping, discovering, creating as well as appreciating patterns and new ideas brought in new focus on mathematisation giving a dialectic relationship to Mathematics to its daily use and opened space to discover Mathematics. Focus also shifted to developing concepts and new algorithms and learners' own ways of solving problems.

Since the NCF 2005, there is talk without clarification of making the Mathematics class-rooms constructivist. Constructivism in classrooms cannot mean that children rediscover all knowledge or form curriculum. The school Mathematics has to be suitable and useful for the stage of the children. Constructivism can shape the manner and pace of transaction within overall goals and expectations. Besides classrooms are not about individual children but about collective teacher assisted learning. Constructivism here means for each child space to think, formulate ideas, descriptions and definitions. A conceptual structure for the child that is open to change with challenges and new situations.

Constructivism does require that Mathematics classrooms consider learners as naturally exploratory, keen to learn and act. They need tasks to stretch their minds and challenge logical abilities, through discussions, planning, strategizing and implementing them. The Mathematics classroom should not desire blind application of not understood algorithm or one way of solving a problem but suggest many alternative algorithms and expect learners to also find new ones. Problems with scope for many different correct solutions must be included to develop nuanced understanding of concepts. Class room must involve all children and give space to do things at their own pace and in their own ways. Besides, children need opportunities to solve problems, reflect on solutions and examine the logical arguments provided to evaluate them and find loopholes. Learning Mathematics is not about remembering solutions or methods but knowing how to solve problems. Problem solving provides opportunities to think rationally, understand and create methods motivates students to become active participants and not passive receivers. This can help learners abstract, generalise, formulate and prove statements based on logic. In learning to abstract children would also need some concrete materials, experiences and known contexts scaffold to help them. In Mathematics we need to separate verification from proof and explanation from exploration and recognising that.

Mathematics not only helps in day-to-day situations, but also develops logical reasoning, abstract thinking and imagination. Enriched understanding developed through it helps us deal with abstract ideas in other subjects as well. These two dimensions have

implications for the resources to be used and the way of using them.

6. Ideas on Resources in the Mathematics Classroom

The idea of materials in Mathematics classrooms has become fashionable. While, the nature and structure of classrooms, remains unchanged, terms such as 'engaging materials' and activities are widely used. The need to interrogate the nature of materials is more for Mathematics as it is not empirical or experimental. Empirical proofs like showing by measurements that the sum of the angles of a few triangle is 180 degree do not constitute a proof. Mathematics being abstract expects us to separate objects of Mathematics from their context. For example, numbers are independent of objects and the unit-ten-hundred system and operations on it not limited to bundles and sticks, place holder cards or any other models.

Resources for Mathematics teaching must align with the purpose of Mathematics teaching, nature of Mathematics, assumptions about the learner and teaching learning process. Printed materials and other concrete objects are examples of materials but, their use is different in Mathematics.

The other important point is that almost everything around us can be used as a concrete material to support learning of Mathematics. Stones, flowers, leaves, water, etc can help quantification. Some objects can aid in visualisation when placed in different positions at different angles, ways and then trying to anticipate how they would appear when changed. The materials presenting models as scaffold, help build foundations of mathematical ideas. Obviously, the use of these has to be linked to the textbooks, worksheets, etc.

It also must be recognised that the nature, specific purpose and use of the materials evolve as we move to different stages of learning Mathematics. This movement may not be entirely linear, but as we go towards building of mathematical ideas the nature of what may be constituted as a concrete experience changes and slowly the requirement and limitation of using concrete experiences to describe mathematical ideas starts becoming evident. Materials and concrete contexts are pegs to create temporary models and to help visualise and manipulate mathematical objects in the form of their concrete models till learners can do without them and deal with mathematical concepts without mediation.

As Mathematics develops hierarchically the earlier learnt abstract ideas become concrete models for further abstraction and formalisation. Base 10 system becomes the concrete model for generalised representation system in any base. Numbers and operations on them generalise to algebra. Geometrical ideas generalise from objects to nets to representations as faces and edges on two dimensional surface and their co-ordinates through point, line, plane, etc. The 3 D system, physically visualisable, moves to a n-dimensional system that has to be mentally visualised. We now look at some aspects of Mathematics and resources for it and different levels.

a) Teaching Mathematics in primary classes

We know that conversations around matematical concepts with opportunity for the learner to express and get feedback is useful for development of thought and conceptual structures. In addition concrete experiences and memories can form the basis of generalization and abstraction.

These are useful whenever new ideas are introduced but, are critical for the primary stage. They change according to the concept and the maturity of the learners and in primary classes would be informed by the struggle of children to read and use the textbook. One point in that is the language may differ from the common languages of learners, but there are difficulties with the way arguments are formulated as well. This implies an important role for language to help children learn concepts and more Mathematics.

In the absence of prior, familiar, easily usable and available abstract conceptual structures with the child, concrete models are critical for the primary classes. The recognition of Mathematics as a discipline emerging from some basic axioms and assumptions based on logical procedures requires concrete experiences and learners engaging with classification, matching, counting, etc. For this they may use their bodies and parts, stones, leaves, any other object in the classroom, games, scores, etc. as means. Dice with a Ludo or a snake and ladder board gives many such experiential opportunities. Pictures can be used for various comparing and matching tasks, identify groups that have more objects than the others. Each of these examples has a different degree of concreteness. However, for Mathematics 5 chairs represent 5 as well as 5 stones can. Five, is just the name of an idea relating to certain other ideas in a specific way. Concrete representations need to use different models for the varied situations and nuances of the concepts.

Promotion of thinking, exploring answers, their comprehension and analysis would not expect or follow from a blind application of un understood algorithm and should encourage children to find many different ways to solve problems. It must point out that many alternative adapted working algorithms and strategies exist and problems can have many different correct solutions and their analysis. The classroom must involve all children and give them space.

The obvious question is, can this be possible? The elements that are required have been mentioned above. To put theses together, the teacher needs to have scope for choosing what she needs to do and is capable of and then ask how she would proceed to fulfil above suggestions. Some suggestions as examples are: Divide grade 2 children into groups of 6-8 with one or two dice and a pile of stones for each group. Each child throws the dice and picks up stones on each turn to see who has more after 5 turns. They slowly abstract that two numbers together produce a third and relate to these and other numbers as abstract entities.

Similarly fractional numbers children can have many relevant examples from daily experience using unequal and equal parts of a whole. They can list and consider these. For example, including concrete models where the fractional number is more than one whole (e.g. 3/2) and recognising that the nature of whole is important.

Experiences of spatial relations, transformations, symmetry, congruence, patterns, measurements through opportunities to play with shapes, etc. build foundations for further development of ideas. The tasks for this can include building shapes, arranging them, observing and anticipating transformed forms, using themselves and surroundings as data sources for organisation and presentation of simple data using for example counters. In all this we know some children start enjoying the play with abstract entities sooner than others. Also different

children enjoy play with different abstractions though all eventually began to use logic and understand Mathematics.

b) Teaching Mathematics in Upper Primary classes

Upper primary Mathematics is linked to experience, but moves further towards abstraction. Children yet need context and/or models linked to their experience to find meaning but eventually must work just with ideas. This challenge is to engage each child through context and move her from this dependence. So while the child should be able to identify principles useable in a context, she should not be limited to contexts. At this stage they may be asked to build their own models and use them as supports.

This stage links the more concrete and direct experience linked with Mathematics in primary classes to formal, less experience dependent abstract secondary Mathematics. This stage must acknowledge that many people after school would take different occupations, most not requiring formulae and algorithms. What all do require is mathematisation of understanding, a deep understanding and appreciation of Mathematics to sharpen analysis and maintenance of logical thread in thought.

Ideas like negative numbers, generalised fractional numbers, rational numbers, letter numbers, ideas like point, line, etc. introduced and developed in upper primary are not easy to model. The representations and examples used to introduce these can be confusing and are inaccurate unless dealt with care and with changing examples with including warnings about limitation and inherent inappropriateness of models.

Through upper primary to secondary classes, these models have to be discarded giving way to the characteristics of Mathematics. The concepts no longer be imbued with materials and tasks that are also different. The empirical and measurement aspect giving way to logic and proof. Concept building now would require more dialogue and at most consciously temporary modeling. The problem formulation and solving must expand here. Not remembering solutions or methods, but knowing how to solve problems, thinking rationally to create methods as well as processes. This motivates and makes active participants to construct knowledge rather than being passive receivers. Problem solving requires students to select or design possible solutions and revise or redesign the steps, if required. These are thus essential parts of the Mathematics classroom program.

7. Mathematics Lab and Beyond

a) In talking about resources and materials there is a lot of talk about Mathematics laboratory. It is important to better understand the purpose of materials in learning Mathematics and the notion of lab so that we use the possibilities in an appropriate manner. We know learning requires experiences related to the concepts being learnt, but Mathematics deals with ideas that are eventually with abstract ideas. For example, numbers are not related to the objects that are used to represent them, a function not related to the curve that depicts it, a triangle that has points of zero dimension and lines of zero thickness can only be visualized in the mind, etc.

With this and the recognition that Mathematics relationships are not empirically provable or verifiable means the the purpose and scope of Mathematics labs need to be sharpened. To illustrate, no amount of measuring angles of quadrilaterals can convince anyone that the sums of 4 interior

angles would always be 360 degree. No model constructed would be free of experimental blemish to show exactly 360 degree. But this does not lead us to conclude that such figures do not have interior angles with sum equal to 360. So what is the purpose of materials in Mathematics learning? This example and question is true for all such uses of Mathematics lab including the oft used and quoted verification/demonstration of Pythagorous theorem

(b) This does not mean that there is no use for materials in Mathematics. They are for many purposes and stages not just useful, but essential in helping learners deal with abstract ideas initially and concretely visualise them. Materials in Mathematics learning while initially helping children experience the abstract ideas concretely have to be withdrawn eventually making the child constructed them in the mind and move away from concrete examples. For example starting from an angle or a ring as a model of a circular shape to a circle drawn on paper, we go through different stages of concreteness in the depiction of the idea of a circle, which can be only imagined as a shape bounded by a line of zero width. When we draw a chord and find the angle subtended at the centre we are dealing with representations of lines and angles. Representing a general circle by a diagram is crucial to understanding of the proof of statements about chord and other properties of a circle. In these we are not taking circles with specific lengths of radius and length of chords, but the generalized abstraction relation. In primary classes we encourage the use of a lot of concrete materials and this usage must drastically reduce through upper primary to secondary classes.

Learners may use these representations, but not see them as being mathematical objects themselves. Therefore, in the secondary classes, Mathematics lab can only provide opportunity to help children concretely visualise some of the ideas to which they have not been exposed earlier. Over emphasis on materials and expecting their use to prove statements can be extremely misleading and become a barrier for an appropriate understanding of Mathematics. The tasks in the so called "Mathematics activity room" therefore must help children explore ideas and start dealing with them in more abstract forms.

It has been pointed out above that the drawing of any geometrical shape is a model and a representation. A circle is not the line as drawn, but the locus of points equidistant from a point in a two dimensional plane. The plot of a function itself, seems far from a concrete reality, but is actually a representation of the more abstract relationship. The plot displays how the function behaves and shows its form.

(c) One example of a resource with significant possibilities is Geogebra. This is different from the usual as it allows creative exploration of graphs and curves as models of functions and use geometrical diagrams to represent and explore relationships. For example marking equal sides and angles, seeing symmetry, transformations and congruence, even constructions of shapes, etc. can be explored through Geogebra. It does not demonstrate but allows the user to model what she wants to explore. Its effective use becomes possible only when the user recognises the abstract nature of Mathematics and uses modelling through Geogebra recognise patterns and reach generalisations. Another example is of using coins, dice or coloured balls etc. to set up an experimental distribution of outcomes helps build understanding of chance, independence of events, probability, etc.

The Mathematics lab needs tasks that make learners explore ideas. The word itself denotes exploration and curious thinking not fixed and correct explanations. Lab therefore has to be multi-dimensional allowing learners to explore ideas and to add to their library of experiences. The focus of the lab must be aligned to objectives of Mathematics teaching, not trapped in explanation and telling syndrome with the recognition that material are temporarily scaffolded to form ideas. All this as a part of the classroom process and not a separate visit to an exotic location called mathematical lab or something like that.

The idea of the Mathematics lab, therefore, has to be in conjunction with the nature of mathematical ideas and questions of what materials, till when, for what and how they should be used need to be considered.

(d) Text Books as a resource in Mathematics

For organised transaction of knowledge along a certain syllabus, including content, abilities and perhaps dispositions as well, textbooks are a necessary evil. The text books in Mathematics are reputed to be dreary and unattractive as they are full of numbers, letter numbers, abstract geometrical shapes, unusual brackets and symbols interspersed by terse sentences. Their standard format suggests that learning mathematical ideas is about seeing examples and then following them to do similar things without conversation or dialogue. Context and experience is a post concept development application or a mere entry point.

The textbooks of Mathematics have to act at a major resource for the teachers to create engaging processes for learners. It has to indicate how to bring and use the experiences of the children, their mathematical ideas embedded in their own language, culture and their daily activities, and make the classroom inclusive, participative, exploratory with simultaneous focus on conceptualisation, formulation and articulation of ideas.

Secondly for the textbooks to be used in the spirit intended appropriate guidance, support and enabling ambience for the teachers and the learners has to be available. It should be able to struggle with the notion that Mathematics classroom and text book are needed for examples of solved problems with methods, techniques, short cuts and memory devices with guidance on how to use and replicate them. The textbook would be organised assuming that since Mathematics is hierarchically organised, learning would be organised similarly and once a topic has been covered, it can be revised by doing problems similar to the ones done earlier.

In the alternative perspective of Mathematics textbook must help the learner engage with mathematical ideas in different ways and experience the nuances. The materials should help elaborate and interlink her concepts embedding them in her language and experience. Hierarchical nature requires spiralling not linear sequence for concepts to become internalised. Learners must come back to the ideas explored on multiple occasions in different contexts and in alignment with different concepts.

The following note to the teacher from the textbook of the NCERT exemplifies not just the way the book is intended to be used, but also the way it has been organised;

"We have tried to link chapters with each other and to use the concepts learnt in the initial chapters to the ideas in the subsequent chapters. We hope that you will use this as an opportunity to revise these concepts in a spiraling way so that children are helped to appreciate the entire conceptual structure of

Mathematics. Please give more time to ideas of negative number, fractions, variables and other ideas that are new for children. Many of these are the basis for further learning of Mathematics. For children to learn Mathematics, be confident in it and understand the foundational ideas, they need to develop their own framework of concepts. This would require a classroom where children are discussing ideas, looking for solutions of problems, setting new problems and finding not only their own ways of solving problems, but also their own definitions language they can use and understand. These definitions yet need to be as general and complete as the standard definition."

The indication is explicit on spiralling, on the need for greater time, multiple contexts and nuances for internalising mathematical ideas and the way the classroom conversation architecture should be organised as well as the expectations from what children would achieve in this process.

As Mathematics in school moves across grades its structure and organisation must be such that the same ideas do not occur concentrically.

The textbooks and other materials must require Mathematics teacher to think and reflect on her class-room experiences and not move mechanically. Such textbooks also call for reasonably long and well-structured orientation program. They should pool new ideas and develop new activities to supplement the textbook.

The language used and the nature and extent of illustrations in it help reduce terseness of the textbook, making it comprehendible for the learner. The flow of the book must aid the learner to pause, reflect and engage with it. It must expect the learner to articulate ideas, concepts, explanations, generalisations, definitions and attempt to prove or disprove mathematical statements. The nature of the Mathematics books have started changing recently particularly after the NCF 2005, both at NCERT and in some states however, many remain in the earlier framework.

Some of the books also have illustrations that show children engaged in doing Mathematics differently. They are shown using resources, chatting and discussing with each other, workout, exploring, imagining and visualising.

The way the textbook is to be used and the nature of the classroom comes out well from this excerpt form the NCERT book *"There are many situations provided in the book where children will be verifying principles or patterns and would also be trying to find out exceptions to these. So while on the one hand children would be expected to observe patterns and make generalisations, they would also be required to identify and find exceptions to the generalisations, extend patterns to new situations and check their validity. This is an essential part of the ideas of Mathematics learning and therefore, if you can find other places where such exercises can be created for students it would be useful."* The key points here being that children are expected to explore, think and work out the answers to the problems. It expects teacher and childrens to create exercises and suggests that the teachers should look for more places for problems that could be located, created or found.

(8) Summing Up:

Mathematics subject in secondary classes includes elaborating and consolidating the conceptual edifice, to make logical and organised arguments, precisely and concisely formulate ideas, to perceive rules and generalization and found mechanisms to prove them. Go beyond numbers to understanding abstract number systems, their properties and general rules about them and similarly in other areas.

Gradually a broad and tenuous agreement on the universal purpose and scope of concepts for Mathematics in secondary classes has been arrived at. Spelt out in the NCF 2005, it is however, yet to reach the classrooms. With multiple formulations of its implications and approaches for the materials and the classroom architecture and processes there is no consensus on strategy to be adopted. On deeper analysis some of the differences in strategy seem to emerge from basic purpose and perspective differences. The unfortunate linking of a child engaging meaningful program to a confused terminology of child-centered or constructivist program has led to a feeling that a Mathematics or a school program could be evolved based on what children want to do on any particular day with an overdose of materials and physical activity. In a pragmatic formulation of meaningful school Mathematics program constructivism would imply the child space to think in the classroom, formulate her ideas, her descriptions and definitions with an attitude and a conceptual structure that is open to changes when presented with new kinds of challenges and situations.

We know that for many children class X is a means to study further, but in the context of Indian education, the secondary classes are the final year of general education and after this students would go in to different roles. A complete general education requires a rounded up Mathematics understanding and capabilities (not mere skills) that are needed by all citizens. In line with the international thinking, the NCF 2005 has enlarged the scope of this with focus on mathematization to attempt enrichment of the scope of thought and visualisation. The secondary school Mathematics, therefore, on the one hand, needs to focus on the consolidation of the conceptual edifice initiated in the classes 6 to 8, but also take it forward to help child explore wider connection and deeper understanding. The logical formulation and the arguments included in each step along with the precision of presentation is of value to engage with the world in more forceful manner.

To summarise, a much larger number of students are now attempting Mathematics as a part of their secondary program as we push towards universalisation of secondary education. The purposes of teaching Mathematics, the pedagogy for it and the materials for secondary classes has somewhat evolved over the last decade or so, but there is a need to put all this in some framework and much more thinking and clarifying is needed. All this has also brought forth the need for context and resources in the secondary classrooms which were earlier devoid of these. The materials in the class-room would not only be an aid to scaffold introduction, but also of engagement with concepts. Like in the upper primary classes and in fact now much more than that, the students need to be asked to create contexts and resources and present them rather than being given materials to manipulate as would be likely in the primary and occasionally in the upper primary classrooms.

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Giving Meaning to Numbers and Operations in Arithmetic

Abstract

This article discusses with examples how basic abstract structures of Mathematics can be taught with examples from real life situations. The author gives examples of contexts that can be used basic for operations of Mathematics.

Introduction

The subject of Mathematics deals with objects and structures abstracted from general patterns. Numbers and operations on numbers are among the basic abstract structures of Mathematics, but Mathematics also involves the application of these abstract entities and their properties in asking and answering questions about the real world. In fact numbers arise through a process of abstraction, from our actions on the real world such as counting and comparing discrete collections of objects.

In learning about the abstract objects of Mathematics and ways to deal with them, it is best not to start directly with them, but to start with real situations. This is the way Mathematics emerged in History, and is the appropriate approach to give children entry into Mathematics. Many research studies inspired by Piaget and others, have shown the usefulness of starting with real world situations in learning Mathematics. It gives numbers and operations meaning in terms of real or realistic situations and supports the initial learning of abstract objects of Mathematics. Approaches to teaching maths that are prevalent in many schools today may be different from this – they may start directly with abstract entities such as numbers, addition or multiplication and then teach children how to apply these to situations through solving word problems. Such approaches are not as effective as approaches that involve starting from situations, solving context based problems and slowly building an understanding of numbers and operations.

Many teachers believe that in order to solve context based problems children must first be taught how to solve them. For example, young children must first be taught the addition algorithm before they can solve addition problems. Teacher needs to understand that children can and do find their own ways of solving problems. Of course, they may not use the standard method that the teacher has in mind, but children's own ways of solving problems are very powerful starting points for learning eventually more efficient and standard approaches to solving problems.

Let us take an example. Suppose I ask a young child who has learnt counting, how many stones are in my left hand (say 5) and how many are in my right hand (say 3). Then I cup both my hands together with the stones inside. If I now ask the child how many stones there are in my cupped hands, she will find her own way of adding the numbers to answer the question. It will be interesting to a teacher to see how she solves this problem. With insights about how different children approach this problem, the teacher will be better equipped to teach addition. The same

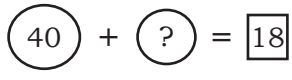
context can also be modified into a subtraction problem. Suppose I have a pile of stones on the floor, say 9 stones. I take away some in my hand. The child counts how many are left – say, 6 stones and tries to guess how many are in my hand. Even if the child has not been taught subtraction, she may find her own ways of solving this problem. Similar examples can be formed for children at all ages.

Although the situation described above is very simple, it can be pedagogically effective. Presenting a child with a situation and allowing him or her to find her own way of solving it can lead to successful solutions by students even for problems that are more complex than the example described above. If a division problem is posed in the context of equal sharing for example, 9-10 year olds may find their own ways of solving the problem. For teachers, choosing the right context and framing the right problem can be very powerful pedagogical tools that give meaning to numbers and operations. Different children may respond to different contexts depending on their experience. If a teacher has a well organized example space of contexts and situations, she will be able to adapt them according to the needs of particular groups of students. In this article, I'll try to present frameworks that describe the kinds of contexts that can be used for the basic operations of arithmetic.

Addition and subtraction

Researchers have suggested that situations involving addition and subtraction are basically of three kinds – combine, change and compare. In the *combine* type of situation, there are two (or more) groups or collections that are either brought together or thought of together. These might be men and women in a group photo, those who are seated and those who are standing, or children and adults. The situation may not involve groups actually coming together , but only thought of together, say the combined population of two adjacent villages, or the total number of motorcycles produced in two different plants of a company. A simple schematic diagram that represents the combine type of situation is the following:

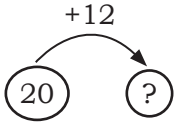
The diagram may even be presented to children along with suggested words (example notebooks, textbooks) and they may be asked to form questions. The child, for example, may form the question: There are 10 notebooks and 8 textbooks in my shelf. How many books are there in all? Note that the diagram can be modified in a way that the question is changed , but not the situation.



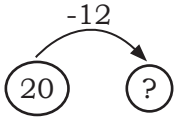
In this case, the question would be: There are 10 notebooks and some textbooks in my shelf. Altogether there are 18 books. How many are textbooks? The unknown here is not the sum , but one of the addends. This can be thought of as a subtraction problem. However, the way in which the child actually solves the problem may be similar to addition, for example she may find the answer by counting up from 10 till 18 is reached. The diagram can be modified so that the question mark is placed in the first circle to yield a type of problem known as "start unknown" combine problem, which is known to be more difficult for young children than the unknown addend problem shown above:

The *change* type of situation involving addition and subtraction involves the

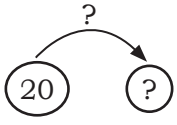
increase or *decrease* in some quantity or number. Of course, the combine and change type of situations are sometimes difficult to distinguish – this is not a hard and fast distinction. An example of a change situation is "There are 20 people in a bus and 12 more people get in". This situation can be represented by the following schematic diagram.



If the number was to *decrease* (12 people got down from the bus), then we could represent it using the following diagram.

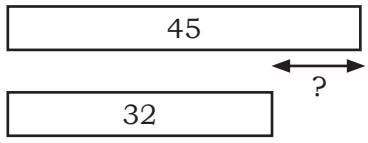


While the problem now looks like a subtraction problem, even the earlier "increase situation" can be changed from an addition into a subtraction problem by changing the position of the question mark as follows:



Indeed, by changing the position of the question mark in each of the diagrams above (there are three possible positions), we can get different problems corresponding to the same context. It is an interesting exercise to think of problems corresponding to each type. We invite you to construct such problems and to try solving these problems with children. Of course, it would be even more interesting to invite children to form their own problems based on the diagrams and the variations.

Finally the compare type of situation can be represented by the following diagram.



A situation that corresponds to the diagram above is: I have 45 story books with me and my friend has 32. How many more books do I have? One could also frame the question as "how many books less (fewer) does my friend have?" Another way to frame it is as an equalization problem: "How many books should my friend get so that we have equal number of books?" For younger children, researchers have reported that equalization problems are easier to solve than comparison problems.

Again, we can see that the diagrams may be modified to place the question mark in different positions to yield different questions. The vocabulary used can also be varied ("more", "less", "equal", "the same as") to formulate different questions.

Kinds of numbers

In the examples that we discussed above, the numbers were whole numbers and the situations involved discrete collections of objects that can be counted. As children grow older, they learn to deal with situations involving not only counting, , but also measurement. With measurement we also move beyond whole numbers. The measurement of common attri, butes such as length, weight, volume, time, monetary value, etc., begins by choosing a unit and producing a measure in terms of multiples of the unit or parts of the unit. When an attri, bute or a part of the attribute is less than the unit, then the unit needs to be subdivided and applied to the part in question. This requires us to go beyond whole numbers to fractions (or positive rational numbers). Thus

the situations of combine, change and compare can also involve quantities that are continuous measures and not just discrete quantities. However, the same framework applies without much change.

A second jump occurs a little after children have been exposed to fractions – that is the introduction of negative numbers. One of the difficulties that children face is in interpreting negative numbers. What does "–2" exactly mean? There are three broad senses in which negative numbers, or more generally signed numbers (positive, negative numbers and zero), are interpreted in situations.

1. **As a state**: We can specify the state of something we are interested in using signed numbers, but only when it is meaningful to talk about positive and negative states. Some examples are temperature of water in a freezer, height above and below sea level. (You can try to think of more examples.)
2. **As a change**: Signed numbers can denote change along with the direction of change: increase or decrease, movement up or down (or forward and backward) or positive or negative growth (for example total annual sales of a company). An interesting example that some teachers suggested in a workshop was to make a table of the weight gained (i.e., change in weight) by a baby every week. (What does a negative change in weight indicate?)
3. **As relation between numbers or between quantities**: An example that illustrates this meaning is the following: Think of a pilot in an aeroplane circling near an airport with many other planes in the air, all waiting to land. The pilot would be interested in knowing the relative height (or altitude) of the closest plane with respect to his own plane. The relative height may be positive (indicating that the closest plane is above the pilot's plane) or negative (indicating that the closest plane is below the pilot's plane).

A more detailed discussion of the kinds of situations in which signed numbers can be applied is possible and very investing, , but it is beyond the scope of this article. We will just note that it is possible to think of addition and subtraction problems involving signed numbers in ways similar to whole number contexts discussed above. That is, we can have combine, change and compare type of situations for signed numbers, in which the signed numbers themselves take on different meanings of state, change and relation. Some combinations of situations are more natural and some are more contrived (Vergnaud, 1982). We will leave this again as something to be explored by the reader.

Multiplication

When we think of situations where numbers are multiplied, a striking difference from addition emerges. When we add two quantities, these quantities are of the same kind and in the same units: we can add two lengths in meters or two amounts of money in rupees, etc. We cannot, of course, add 5 meters and 10 centimeters to get 15 metre-centimetre – before adding we must convert one of them into the same unit as the other. However, when we multiply two numbers, it is very rarely that they are quantities of the same kind.

The most common kind of situation involving multiplication can be represented by the following schema:

Rate × *Quantity 1* = *Quantity 2*

To take an example if a kg of potato costs Rs 25, we can find the cost of 5

kg using multiplication:

Rs 25 per kg × 5 kg = Rs 125

Note that in this example all the three quantities are of different kinds and in different units. The first is an amount of money per unit weight, that is, a rate, the second is weight and the third is amount of money. Most multiplication situations fit this pattern. You can try to make up situations where the quantities involved are length, time, volume, etc. and are combined in various ways. Two other types of situations involving multiplication are related to the rate type of situation, , but are slightly different. These are situations where quantities are scaled up or down by a scaling factor (for example in maps), and unit conversion problems (how much is 2.3 m in cm?). The scaling factor or the unit conversion factor is similar to a rate, , but involves only one kind of attribute or measure unlike a rate, which involves two kinds of attributes or measures.

There are situations where two quantities of the same kind are multiplied, but these are relatively fewer than the situations described above. In fact, in elementary classes the only example is the multiplication of two length measures to obtain area, or the multiplication of three length measures to obtain volume. (Note that, in contrast to multiplication, if we add two lengths, we only obtain another length.) Another uncommon kind of situation is where two quantities are multiplied, where neither is a rate. This occurs in school Physics – in the case of a lever or a balance, we multiply length and weight to find the moment about a fulcrum or pivot.

The situations described above involve measures that are mostly continuous measures. Situations involving multiplication of only whole numbers are even simpler. They usually involve finding the total quantity of equal sized collections of discrete objects, like 12 boxes with 10 eggs each. There is an interesting type of situation of multiplication of whole numbers that is different from these. It involves finding what is sometimes called the cartesian product of two sets. For example, if I have three shirts and four trousers, then how many different combinations of shirt and trouser can I wear?

A final remark is on the question of which types of multiplication problems are more difficult for children to solve. This does not always have an easy answer like situations of type A are easier than situations of type B. Of course, some situations such as equal groups of discrete objects are simpler because they can be modelled by children using icons, objects or even mental objects. However there are many factors that make a problem relatively easier or more difficult – the familiarity of the situation, the language and vocabulary in which the problems are posed, how big the numbers are, what type of numbers they are, the relation between the numbers, etc. As a teacher works with particular groups of students, by varying the situations, a teacher will develop an understanding of which problems are easier and which are more difficult. Even better, teachers could form a group and try out different variations of a problem and share and discuss their findings with each other.

Division

Division is the inverse operation of multiplication. So division situations are related to multiplication situations. However since multiplication commonly involves two kinds of quantities that are multiplied, division can be interpreted in two ways. Let us consider first only multiplication of numbers

25 × 5 = 125

We can have two division facts that are the inverse of this multiplication:

125 ÷ 5 = 25 and 125 ÷ 25 = 5

Let us interpret this now in terms of the situation that we considered above – the cost of 5 kg of potatoes. We could have two kinds of division problems corresponding to this situation: (a) If the cost of 5 kg of potato is Rs 125 than what is the cost per kg of potato and (b) If the cost per kg of potato is Rs 25, then how many kg can I buy with Rs 125? We see that in the first case division is used to find the rate or per unit cost and in the second case division is used to find the number of units.

Even in the case of division involving only whole numbers representing discrete quantities, we see that there are two meanings of division that are similar to the two meanings of division above. Let us take the example of 8 ÷ 2. (Note that this is a different starting point from the previous example where we started with one multiplication fact and two corresponding division facts. Now we start with only one division fact and discuss two different meanings.) We can interpret 8 ÷ 2 in two ways.

On the left the division shows the number per group when the number of groups is given to be 2. On the right the division shows the number of groups when the number per group is given to be 2. The situation on the left corresponds to the "equal partitioning" meaning and the situation on the right to the "equal grouping" meaning. The equal partitioning meaning corresponds to finding the quantity per unit in the measure context, and the equal grouping meaning to finding the number of units given the quantity per unit.

The remarks above may seem to be rather trivial and obvious. However, knowing the different meanings of division becomes useful when dealing with a question like the following.

- Construct a word problem that corresponds to the operation 1¾ ÷ ½ .

This is a famous problem that was given by the researcher Liping Ma to Mathematics teachers in the USA and in China (Ma, 1999). She found interestingly that almost all teachers from the USA found it very difficult to construct a word problem corresponding to the given division fact. In fact, many teachers suggesting taking 1¾ pizzas and sharing it among two people, which corresponds to the operation 1¾ ÷ 2 and not to 1¾ ÷ ½. Many of the Chinese teachers, in striking contrast, could come up with several examples of situations. In fact the situations corresponded to three different meanings of division as seen in the examples below:

1. If a machine can lay ½ km of road in one day then how many days will it take to lay 1¾ km of road. (This corresponds to finding the number of units given the quantity per unit, similar to the equal grouping meaning for whole numbers)

1. A wealthy man is partitioning his farm to distri, bute it among family members. Different family members get different shares. If half a share corresponds to 1¾ of an acre, then what is the size of one share. (This corresponds to finding the quantity per unit, given the number of units, which in this case is half a unit.)

1. If the area of a rectangle is 1¾ units, and its length is ½ unit, what is its breadth? (We have not discussed this meaning in the context of division, , but we have discussed it in the context of multiplication. It corresponds

to the situation of product of measures – length × length = area.)

Conclusion

We have discussed the different kinds of situations that correspond to the arithmetic operations of elementary school and tried to evolve a framework or a categorization of such situations. How is this useful? Firstly, situations give meaning to numbers and to operations. This makes it easier to learn and deal with abstract entities like numbers and operations. Teachers need to have the capacity to design situations and flexibly adapt them to their classroom teaching. I hope that having a synoptic overview of kinds of situations will help in designing appropriate and powerful contexts for learning arithmetic.

When one moves beyond whole numbers to fractions and integers, the range of situations and meanings expands. It is important as well as challenging to design appropriate situations that involve operations with fractions or signed numbers. How does one design a context which is modelled by the multiplication fact (–3)× (–5) = +15? A discussion of this challenge will need one to delve deeper into the meaning of integers, which we will not be able to do here. Some discussion of this and related issues concerning integers can be found in Kumar, Subramaniam and Naik (2015). Similarly, fractions also have different meanings (sometimes called subconstructs) in different contexts (Naik and Subramaniam, 2008).

Further, developing an understanding of how numbers connect with real situations makes one sensitive to what is an appropriate use of number and operation and what is not. This can be illustrated with the help of a puzzle that was recently circulating on social media. Here is the puzzle:

I had Rs 50 and I went shopping. Here is what I spent:

Spent (Rs)	**Balance (Rs)**
20	30
15	15
9	6
6	0
50	**51**

The puzzle then asks: where did the extra Rs 1 (in Rs 51) come from?

The resolution of the puzzle consists in realizing that while it makes sense to add the numbers in the left column, it does not make sense to add the numbers in the right column. The numbers in the left column are all amounts of money which are distinct, non-overlapping parts of Rs 50 and together make up Rs 50. The numbers in the right column and not distinct parts, , but parts of Rs 50 which are contained in other parts and cannot be added to represent a whole (i.e., Rs 50). For example if I spent only Rs 1 for the first two items that I bought, the first two rows in the right column would be Rs 49 and Rs 48. If you add them it is much greater than Rs 50! This is because Rs 48 is already contained in the earlier balance of Rs 49. In this example, it is rather easy to find out what has gone wrong and why it is inappropriate to use addition. There are many subtle ways of fooling people using numbers. One of the goals of Mathematics education is to be able to see through such incorrect uses of Mathematics.

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What Mathematics Matters to Teachers?

Abstract

This article draws attention to the need of mathematising the teachersof mathematics. The paper questions and seek answers to the didactical approaches that should be adopted to engage teachers in acts of thinking mathematically.One of the proposed ways is by challenging teachers' existing mathematical cognition in a constructive manner. The paper further elaborates a task that was instrumental in setting up conditions for thinking, reasoning and making conceptual connections.

When the NCERT's National Focus Group took a position on teaching of Mathematics (Position Paper on Teaching Mathematics, 2005), they explicated the higher and narrower aims of learning Mathematics, emphasising teaching to be central around 'mathematising' the child's mind. Indeed, undoubtedly the document, though written perspicaciously, falls short in defining the knowledge, skills and preparation required on the part of teachers for meeting such goals. We question if our teachers, both in the service and preparing for the service get enough opportunities to mathematise themselves.

Through this article, I wish to focus on the preparation of Mathematics teachers (through Teacher Preparation Programmes) and of their continual learning (through Professional Development Programmes) as a means of getting mathematised. I question if our teachers are mathematised enough. Do they understand acts that lead to 'mathematisation'? Only when the teachers are exposed to the strategies embedded within the frameworks of promoting mathematical thinking, they will be able to adopt similar routes in their classrooms. My proposal is to question and seek answers to the didactical approaches that we, as Mathematics Teacher Educators need to implement to let our student teachers, who are already familiar with the content matter, approach the subject from a wider perspective, one that is needed for teaching children effectively.

There are many opportunities for educating teachers. At the entry stage, the Institutions such as CTEs, IASEs , DIETs, SCERTs and other teacher training institutes provide the first course on teacher preparation. The in-service, teachers get several opportunities to revive their skills and knowledge through Professional Development Programmes (PDPs). The arguments and proposal that I am going to present in this article hold for all such programmes. I am going to argue that revisiting mathematical content, judiciously, at all levels of professional engagements is not only important, but also challenging for Mathematics Teacher Educators.

I will refrain myself to comment on the content knowledge that the entrants of a teacher preparation program come with. This requires a detailed analysis. However, through my informal observations, while engaging with them, I understand that at times the prior understanding of students who come to the teaching profession is so shaky that in no case it can be rectified in a one or a two-year teacher preparation course. The pre-service teachers, though graduates and post-graduates

in Mathematics, face difficulties in learning and understanding the fundamentals of the school content. The Mathematics department does not teach them this Mathematics and definitely, there is no scope to redo all the required Mathematics in a curt two-year preparation course.

The challenges of in-service teachers are somewhat dissimilar. Teachers being burdened with multiple responsibilities, trying to complete courses within rigidly limited time, being one of them, often leaves them bewildered. Since school assessment is predominantly based on quick recall of correct procedural know-how, the teachers do not feel a need to move beyond certain shortcuts and routine algorithms. I think the teachers also get conditioned over the years. They recognize what is significant (in terms of assessment only) and this limited view sets their teaching objectives. I will not elaborate how such a view harms children's learning as many have expressed this at various platforms. Further, I understand (again through informal talks during PDPs) that as teachers gain experience, they dissuade gaining new insights. What is needed in such programmes is to produce opportunities that evoke a sense of challenge in the teachers. The tasks should urge them to learn the basics, which get missed in the course of teaching concepts repeatedly.

In a nutshell, pre-service teachers have never dealt with the content that is needed for teaching and those inservice feel that looking back at the content is a fruitless exercise. Either way, the call is on rebuilding the mathematical knowledge of teachers.

Different people interpret the nature of the content needed for Mathematics teachers differently. Some consider it as revisiting the school content while for others it means going beyond school Mathematics. What sort of Mathematics do teachers really need to know is undefined or at least unfamiliar to many. What Mathematics matters to them? A knowledge of content is as significant as the pedagogical ways of transacting it. The study of some researchers such as Shulman, followed by Ball, Hill, Adler and their colleagues (to mention just a few) provides a starting point to building teachers' content and pedagogical understanding. They state cogently that preparing for a profession, especially of teaching, requires the amalgamation of content and a judgement of appropriate teaching practices. These researchers extensively elaborated on the tasks which teachers need to engage with and those that can be engrafted in their classrooms.

I go further to say, the pre-service teachers must also be provided with opportunities for thinking mathematically. The opinion is on necessitating teachers to reason mathematically. To bring faith in their practices, the teachers themselves need to be engaged in acts of conjecturing, communicating, reasoning, debating and making connections. It's akin to making teachers as agents of creating Mathematics. In other words, teachers need to make sense of the mathematical activities themselves.

One of the ways through which I try to encourage my students to (and teachers in PDP workshops) revisit the content is by 'challenging their cognition'. The didactic intention is to present the content that questions their mathematical knowledge in a constructive manner. I understand that it is not easy to change teachers' beliefs and methods. It calls for shaking their existing beliefs, schemas, practices and methodologies. Of pertinence here is prefixing the verbs of learning with 're', re-look, re-learn, re-conceptualise and re-engage with the concepts. The

teachers should be afforded with the same experiences as their students are likely to get when they come across a new concept, process or idea. Pre-service and in-service teachers should get opportunities to re-learn and re-conceptualise the Mathematics. The mere presentation of school mathematical content or just good examples of teaching does not avail. Create situations that make unfamiliar. Teachers' training in the content should be an exercise of creating conflicts in their existing cognition. The didactic approach should try to capture, to some extent, the issues that make learning meaningful and which arouse alertness to students' difficulties.

Several arguments support the proposed didactical approach. To begin with, teachers need to be aware of the learning gaps and difficulties that their students face when they come across a new idea. That is, teachers need to view Mathematics from the same platform as their students do while learning a new concept. This is possible when new experiences are provided to the old content. Secondly, teachers' own experiences of learning Mathematics are eclipsed by procedural approaches. By challenging teachers' existing practices a strong need to rivet on the underlying conceptual knowledge would emerge. Lastly, it is hoped that by delving into such activities, teachers would become better observers of their students' work.

Designing appropriate tasks through which the content and pedagogy understanding can be amalgamated is not easy. While designing such tasks my intention is to engage teachers (and students) in reflective processes. In this article, I'll present one such exercise where my students (and teachers) got opportunities to re-look at a concept as a content building exercise. The example draws learnings from integrating the history of the evolution of mathematical concepts. The tasks set up the conditions for thinking, reasoning and making conceptual connections. The illustrated example is one of the many instances of a more general didactic approach.

Using the History of the Evolution of Concepts

In this section I will describe a lesson that integrates the journey of number formation, starting from the fundamental acts of enumeration to systematic approaches of extending numbers and creating Numeration systems.

To start with, I frame a set of inquiries that provide me with a lead: What is it about Numeration systems that the teachers need to know? What are the central characteristics of the current Numeration system? How did humanity arrive at these fundamentals? What aspects does a teacher need to be alert with while building the idea of the place - value system? Which pre-concepts are needed? These questions helped me in picking out moments of importance in the evolution of Numeration systems. Though the above thoughts assisted in planning for teachers, they are, to a large extent, guided by the learning difficulties that children face while working in the place-value system.

Indeed, I have to think of the areas that would kindle teachers' attention. Presenting history could at times be dull and boring. The concern was to present story as a concept building exercise. Conscious efforts were made not to present the material in a chronological way or as biographies (of mathematicians, as presented in textbooks). It was ensured that the concepts bind themselves to form a meaningful sequence, rather being presented as disjointed chunks of information. Thus, instead of

approaching History as a chronology of events, the tasks were arranged to bring out a conceptual-chronology. Moving from the easiest to the complex numeration systems. The intention was not a mere transmission of the historical facts. The material should let teachers construct their ideas and make sense of the fundamental processes that form a numeration system. It was kept in mind that during this course the teachers should be able to deduce linkages between the various numeration systems. Concomitantly, the readers also had to bring out the shortcomings of a numeration system and the determinants that led to the systematisation of others. This requires a reflective mode, which is also flexible. The teaching materials were designed to evoke reasoning.

The Material

To understand the basis of the current Numeration system, i.e. the Hindu-Arabic system, two fundamental concepts need to be established: the positional feature that assigns value to a number and the base number that forms grouping. The journey of arriving at these two concepts is elucidated through the modules.

The first set of module presents the genesis of counting as a need for keeping records. The first set of worksheets introduces the reader to the primitive ways of counting. The objective is to evoke students' attention to how humans made connections with the principle of one-to-one correspondence and used the ready-to-refer material such as body parts to do so. The module also brings to the fore the idea that in the early stages the concept of numbers did not imply nor was there any necessity to have them. The thought of expanding numbers evoked the need for grouping. This idea is brought up in the next set of worksheets. In this set, the transition to Numeration and number words is covered. As part of a reflective exercise, the students have to deduce mechanisms of expanding numbers beyond finite counters. This exercise evokes a need to systematise the expansion of numbers to extend them to infinity. This exercise culminates in a reflective discussion elucidating the advantages of creating a Numeration system over eNumeration.

For a more formal growth of numbers, numeration systems emerged. The second set of the module covers the numeration systems of five influential civilisations: Egyptian, Babylonian, Roman, Greek and Mayan, in this order. The module presents an elaborate description of the Numeration system of these civilisations.

The structure of the worksheets on numeration systems is similar. Each module comprises an introductory phase and a chart demonstrating Numeration system of a civilisation. The students have to study the system and conjecture the rationale behind the processes that would have contributed to the idea of systematising the process of Numeration in the given civilisation. The students are encouraged to find the grouping number, basis of grouping and the symbolic representations of the numbers of the respective civilisation. To give them hands-on experience, they are encouraged to construct numbers abiding to the rules of the system. This exercise acquaints the readers to the rules and syntaxes.

As part of the next exercise, the students have to work on basic calculations of addition, subtraction, multiplication and division on the numbers they had formed during the previous exercise. This activity often turns out to be quite hard one. Working on a new system is challenging. It's not easy to forget the old rules and learn new ones. We have been conditioned to

working in the Hindu-Arabic system, so our habituated mind dissuades learning the new format. Performing calculations on the new system challenges the existing cognition. While playing on the arithmetic operations the teachers (and students) face, for the first time, situations that shake up their existing knowledge. It draws their attention to the processes that people of particular civilisations could have adopted. Later, after much practice, as a follow-up, they are asked to develop the algorithms for calculations. At this point one sees teachers reasoning, making arguments, conjecturing the possibilities and convincing their colleagues on the key features of the concept in hand. All acts that lead to thinking mathematically are evident in these sessions. The activity deconstructs several ideas and this deconstruction, in turn, establishes a profound understanding of some of the most elementary concepts such as of grouping, selecting a suitable number for grouping and determining the value of a number based on its position. Finally, after achieving enough acumen with a particular numeration system, as a reflective exercise, the students (and teachers) are required to compare the various numeration systems and bring out the similarities and differences. These reflections leave a deep impact on the teachers' (as well as students') learning.

Wherever available the material is also supplemented with videos. The worksheets are made in self-explanatory mode. The participants work in pairs and the results are shared with the entire group. We pause occasionally to share our understanding, seek clarifications and put forward the deductions. The solutions are never stated directly, but are worked out collectively as a group. The whole group discussions give scope for larger debates and consensus. Since none of us know the grounds behind the development of a system, we all make several conjectures, some mathematical, some not-so-mathematical, but interesting.

By-products

At the beginning the material on the history of the evolution of numeration systems was prepared with a prime agenda of challenging teachers' beliefs and methodologies on the current place-value system. We are glad to share that each time this lesson is executed many other ideas and concepts emerge as by-products. Among these, the most frequent one that comes up virtually in all the discussions is about the genesis of zero. Discussions regarding the existence and nonexistence of zero in a numeration system are most frequent. Arguments considering zero as a void or as a placeholder or as a cardinal number emerge invariably and often lead to long discussions and debates. Teachers are seen debating, sharing thoughts, arguing and attempting to provide convincing reasoning to their ideas regarding the role and value of zero. These discussions always come as residual and a much-valued one. I call it a bonus!

Knowing history gives a pretext and a context. When one studies the growth of an idea, one gets to appreciate its emergence as a product of cross-generational and cross-conceptual confluence. This is the second by-product. While walking through the galleries of history, teachers and students try to decode the cultures and social norms of the civilisations. Along with the advancements in Mathematics, the readers also visualise the subject as a cultural product, one that is created by people, at a particular time, attributed to the then existing needs. They appreciate the ever-evolving character of Mathematics

as an enterprise of human minds.

Finally, being open-ended, the activities encourage a research-based learning, giving a mathematical ownership, helping students and teachers profit mathematically.

Some Closing Thoughts

As a concluding comment, I urge the Mathematics Teacher Educators to create situations that mathematise the Mathematics (as Freudenthal calls it) for the teachers. Such approaches, I trust, will serve in preparing teachers to become more serious observers, who are receptive to listening and respecting their children's ideas. By becoming agents in the process of creating Mathematics, the teachers will understand the subtleties of 'thinking Mathematics'. By reflecting on their difficulties, teachers become conscious of the difficulties that their students are likely to face.

I am aware that the assertion I am making is not new to most of the Mathematics teacher educators, yet re-educating teachers in a sense-making activity, appropriate to their level, is a challenge for many, if not for most. Meaningfully mathematising the Mathematics for teachers should be a critical dimension of any teacher education programme. Summing up, we need to create situations for teachers to 'experience Mathematics' which, in turn, will strengthen their mathematical knowledge. It is these choices of experiences that make the subject memorable, one that is cherished lifelong.

Notes:

2. 1. In the B.Ed. programme of the University of Delhi only those who are graduates or postgraduates in Mathematics can opt for the courses related to Teaching of Mathematics. During their internships, the graduates get to teach middle grades and the postgraduates teach higher classes.
2. Lee Shulman pioneered the special kind of knowledge that is required for teaching. He had recognised the dichotomies that existed between content knowledge and pedagogical practices in teacher preparation programmes. He thus promoted an amalgamation of the two. He is credited with popularising the phrase Pedagogic Content Knowledge (PCK) for teachers. Following his ideas, in the field of Mathematics Education, many Mathematics educators such as Deborah Ball, Heather C. Hill and Jill Adler have done extensive work in the understanding, elaborating and categorising PCK as a construct. As a result, the idea of PCK now encompasses many dimensions; some theoretical, some radical, some controversial.
3. Here, 'students' means pre-service teachers (or student-teachers) who are enrolled in the Teaching of Mathematics course of B.Ed. programme, University of Delhi.
4. For a collection of videos on History of Mathematics, visit the BBC website. They hold a collection of documentaries on various civilisations and biographies. My personal favourite on numeration system is their documentary-Story of One. Retrieved January, 2016 from https://www.youtube.com/watch?v=qevpRffg6wc.
5. For more along this idea, refer to Hans Hans Freudenthal's article, Why to Teach Mathematics so as to be useful. *Educational Studies in Mathematics*. May 1968, Volume 1 (1), pp 3-8.

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गणित, पाठ्यपुस्तक और शिक्षक

सार

यांत्रिक तरीकों से गणित सीखने-सिखाने की आलोचना के साथ-साथ यह आवश्यक है कि इससे होने वाली परेशानियों को सुलझाया जाए शिक्षक संबंधित इस लेख में हमने इन कुछ पहलुओं को सामने रखते हुए इससे शिक्षकों की तैयारी व क्षमता वर्धन से संबंधित सरोकारों की चर्चा की है। हमारा मानना है कि गणित शिक्षक के लिए आवश्यक क्षमताओं में एक और क्षमता यथा गणितीय संवेदनशीलता जोड़ी जाए। हमने इस शोध पत्र द्वारा इसे समझाने का प्रयास भी किया है।

अक्सर यह सुनने में आता है कि गणित सभी जगह है यह हमारे दैनिक जीवन के हर पहलू से जुड़ा हुआ है - प्रकृति में, हमारे आस पास और हमारे द्वारा उपयोग किए जाने वाली हर तकनीकी में यह उपलब्ध है। परन्तु बच्चों का अनुभव कभी यह नही कहता कि स्कूल में सिखाया जाने वाला गणित उनके जीवन में उपयोगी है बल्कि वे अक्सर कहते हैं कि गणित एक अलग तरह का विषय है जिसे समझनें में कठिनाई आती है और यह सबसे कठिन भी लगता है। शायद समाज का एक बड़ा वर्ग भी यही सोचता है।

शिक्षा के संदर्भ में कई सारी भ्रान्तियां व्याप्त है, जैसे: विज्ञान और गणित पढ़ने वाले विद्यार्थी कला, वाणिज्य अथवा अन्य विषय पढ़ने वाले विद्यार्थियों से ज्यादा होशियार होते हैं। कई विषय ऐसे हैं जिन्हें रटना पड़ता है, गणित रटने का विषय नहीं है। इन कथनों से शिक्षक और समाज दोनों सहमत लगते हैं। अब बच्चों के पास यह प्रश्न रह जाता है कि गणित रटने का विषय नहीं है और यह समझ में भी नहीं आता तो फ़िर क्या करें? ज्यादातर विद्यार्थियों के समक्ष यह गंभीर समस्या बनकर उभरती है जो कि उनकी गणित के प्रति रूचि कम करने लगती है। कई विद्यार्थी दसवीं पास होने का इन्तज़ार करते हैं ताकि और गणित पढ़ना ना पड़े।

गणित के प्रति समाज का यह दृष्टिकोण गणित के शिक्षकों को समाज में और अधिक मान, सम्मान और प्रतिष्ठा देता है शहरी और अर्द्ध शहरी क्षेत्रें में तो गणित के शिक्षकों का टयूश्न व्यापार भी अच्छा चलता है। अच्छा शिक्षक वही होता है जिसके अनुमान किए गए प्रश्नों में से ज्यादातर प्रश्न परीक्षा में आते हैं। परीक्षा उन्मुखी इस व्यवस्था में गाणित का अध्ययन–अध्यापन एक कर्मकाण्ड जैसा है जहां शिक्षक कुछ सवालों को बोर्ड में हल करते हैं बच्चे उन्हें अपनी कापियों में नकल करते हैं शिक्षक द्वारा दिया गया गृह कार्य कुछ बच्चों द्वारा हल किया जाता है तथा शेष बच्चे इन्हे अपनी कापी में नकल करते हैं। इस प्रकार प्रश्नावली हल करने की औपचारिकता पूर्ण होती है और गणित के सवाल हल करना एक यांत्रिक प्रक्रिया बनकर रह जाती है। इन कक्षाओं में गणितीय सोच को पनपने का कभी मौका नहीं मिलता।

यांत्रिक तरीके से गणित सिखाकर बच्चों की उपलब्धि सुनिश्चित करने की प्रक्रिया में कई गणितीय भ्रांतियां भी उत्पन्न की जाती हैं जिनके कारण उच्च स्तर पर कई परेशानियां उत्पन्न होती हैं, उनमें से कुछ इस प्रकार हैं -

1. तीन अंकों की संख्या सदैव दो अंको की संख्या से बड़ी होगी अर्थात जिस संख्या में ज्यादा अंक होंगे वह सदैव कम अंक वाली संख्या से बड़ी होगी।
2. दो संख्याओं का गुणनफल दोनों संख्याओं से पृथक-पृथक बड़ा होता है।
3. किसी संख्या को 10 से गुणा करने का अर्थ उस संख्या के दाहिने ओर एक शून्य बढ़ा देना है।
4. अकृतियों की एक भुजा हमेशा क्षैतिज अक्ष के समांतर होती है।
5. गुणा में योग की प्रक्रिया बार-बार दोहराई जाती है।
6. 3 लाल पेन और 4 काले पेन में लाल पेन का अनुपात क्या है का जवाब 3:4 होगा।
7. आयत की दो भुजाएं बराबर होती हैं और वर्ग की चारों।

ऐसे कई गलत यांत्रिक नियम लेकर गणित की पढ़ाइ आगे बढ़ती है और आगे इन नियमों के व्यपकीकरण में परेशानी जाती है। ज़ाहिर है कि यह कथन कभी-कभी सच हैं और कुछ तो गलत ही हैं। याकृत संख्या से आगे बढ़ कर ऋणात्मक ही और भिन्नात्मक संख्या तक जाने में संख्या वाले सभी नियम गड़बड़ा जाते हैं। सभी वर्ग आयत भी होते हैं आदि। सभी कथनों को ठीक नहीं माना जा सकता

यदि गणित को सवाल हल करने के यांत्रिक तरीकों से हटकर सोचना है तो सवाल हल करने के निर्धारित तरीकों से हटकर एक ऐसी चिंतन क्षमता विकसित करनी होगी जहां सामान्य स्वअनुभवों के ख़जाने को विभिन्न परिस्थितियों में आवश्यकता अनुरूप समझ के साथ परिवर्तित स्वरूप में उपयोग किया जा सके। इस प्रकार के चिंतन को बढ़ावा देने हेतु गणितीय चिंतन की आदतें विकसित करनी होगी। इस प्रकार की आदतें विकसित करने हेतु निम्नांकित कुछ प्रक्रियाओं पर ध्यान दिया जा सकता है-

1. विभिन्न पैटर्न में छुपे सम्बन्धों को पहचानकर आत्मसंतुष्ट होना। इसका लाभ यह होता है कि, पैटर्न व्यपकीकरण को समझाने में तथा कब संबंध पैटर्न के रूप में नहीं देखा जा सकता है, को समझने में मदद करेंगे।
2. प्रयोग करने की स्वतंत्रता- गणित की कक्षाओं में प्रयोग करने के अवसर अत्यंत न्यून हैं, पाठ्य पुस्तकों में दी गई गतिविधियों के अतिरिक्त और कोई प्रायोग नहीं किया जाता। गणित के अवयवों से जब तक बच्चे नहीं खेलते, इनके साथ विभिन्न प्रयोग नहीं करते तब तक इन अवयवों के उपयोग एवं गुण से परिचित नहीं होते इसलिए यह आवश्यक है कि बच्चे प्रयोग करें, प्रयोग के निष्कर्ष पर संशय व्यक्त करें तथा अपने संशयों को दूर करने हेतु पुनः प्रयोग करें।
3. व्याख्या करने के अवसर- गणित की भाषा, सामान्य भाषा में कुछ संकेत एवं आंतरिक संरचना में परिवर्तन कर बनाई जाती है। इसका अर्थ यह है कि सामान्य भाषा, गणित की भाषा का उप समुच्चय है। बच्चों को गणित की भाषा का उपयोग करने हेतु खेल खेलने तथा कई प्रकार की गतिविधिा करने के अवसर उपलब्ध कराने की आवश्यकता होती है, जिनमें से कुछ निम्न प्रकार हो सकते हैं-

अ- सवाल हल करने की प्रक्रिया में विभिन्न पदों की कारण सहित व्याख्या, अर्थात जो कर रहे हैं उसकी कारण सहित व्याख्या।

ब- सामान्य जटिल भाषा को संकेतों के माध्यम से सरल बनाना। व्याख्या करने हेतु संकेतों को ढूंढ़ने एवं उपयोग करने की क्षमता का विकास करना।

स- तर्क करना- विद्यार्थियों को उनके द्वारा निकाले गए निष्कर्ष के पक्ष में तर्क देने के पर्याप्त अवसर उपलब्ध होने चाहिए।

द- विद्यार्थियों को अपने विचार, परिणाम, अनुमान, तर्क, प्रमाण, प्रश्न एवं राय लिखने के पर्याप्त अवसर उपलब्ध होने चाहिए।

4. बच्चों के पास सुधार करने के अवसर होने चाहिए, बच्चों को एक अनुसंधानकर्त्ता के रूप में कार्य कर संक्रियाओं एवं अन्य अवयवों को बदलकर एवं प्रतिस्थापित कर नई-नई सम्भावनाओं को ढूंढ़ने का पर्याप्त अवसर होना चाहिए। जैसे कि-

   इसे गलत-गलत घोषित करने की अपेक्षा यदि उन्हें यह सोचने का मौका दिया जाए कि किन-किन संख्याओं के लिए यह कथन सही है अथवा क्या संक्रिया के लिए उपयोग किए गए चिन्ह को बदलकर इस कथन को सही बनाया जा सकता है।

5. बच्चों के समक्ष सदैव नए खोज की सम्भावना होनी चाहिए- बच्चों के समक्ष ऐसी परिस्थिति उत्पन्न करनी चाहिए कि वे नई बातें सोच सकें। जैसे कि यदि वृत्त के केन्द्र में बनने वाले कोण का मान 360 अंश के स्थान पर 400 अंश हो तो क्या होगा?

छत्तीसगढ़ में नई पाठ्यपुस्तकों के निर्माण में इन बातों का धयान रखा गया है तथा बच्चों को कार्य करने के कई अवसर भी उपलब्ध कराए गए हैं। पाठ्यपुस्तकें बच्चों को

सम्बोधित की गई हैं तथा इनमें उपयोग की गई भाषा भी बच्चों के लिए उपयुक्त है। फिर भी बच्चों की समझ तथा अधिगम स्तर में कोई बड़ा परिवर्तन नहीं दिखा। जिसका एक कारण तो यह है कि किताबों में रखे गए प्रश्न अभी भी पुराने तरीके के हैं जो यांत्रिक रूप से गणित हल करने हेतु प्रेरित करते हैं और चूंकि प्रत्येक अध्याय के अन्त में अभ्यास हेतु प्रश्न है, इसलिए उस अध्याय को पूर्ण करना अर्थात अध्याय के अन्त के प्रश्नों को हल करना है। इन प्रश्नों से गणित सीखनें में दो तरह से अवरोध उत्पन्न होते हैं, एक तो कोर्स पूरा करने का अर्थ पुस्तक में दिए गए प्रश्नों को हल करना माना जाता है तथा दूसरा इन प्रश्नों के कारण शिक्षक नए प्रश्न नहीं बनाते और परीक्षा में भी यही प्रश्न पूछे जाते है। इस प्रकार यदि एक किताब 10 साल चलती है तो 10 साल नए प्रश्न नहीं बनते। इसलिए यह आवश्यक है कि गणित की पुस्तकों में अभ्यास के प्रश्न दिए जाने के तरीकों पर गंभीरता से विचार किया जाये। क्या यह आवश्यक है कि अध्याय के अन्त में ही अभ्यास के प्रश्न हों? क्या अभ्यास के प्रश्न बनाने का स्थान शिक्षकों और विद्यार्थियों के लिए खाली नहीं छोड़ा जा सकता? क्या पाठ्यपुस्तक से अलग प्रश्नों के संकलन के प्रकाशन के विषय मे विचार किया जा सकता है? इस प्रकार कुछ और भी सोचा जा सकता है ताकि गणित की पाठ्यपुस्तकों को अभ्यास प्रश्नों के हल करने से हटकर देखा जा सके।

दूसरा कारण शिक्षकों की तैयारी हो सकता है। एक गणित शिक्षक के लिए गणितीय ज्ञान का अर्थ गणित का ज्ञान विशेषकर एक विषय के रूप में, गणितीय तथ्य, अवधारणायें, प्रक्रियाएं एवं इनके मध्य आपसी सम्बन्धों के साथ - साथ उन तरीकों का ज्ञान जिसमें गणितीय विचारों को प्रस्तुत किया जा सकता है। उन्हे यह भी जानना आवश्यक है कि गणितीय ज्ञान की उत्पत्ति कैसे होती है, गणित की प्रकृति तथा मानदण्ड एवं मान्यतायें जो गणित के प्रमाणों को जांचने के लिए आवश्यक हैं। गणित का ज्ञान के प्रचलित अर्थ में गणित शिक्षण के लक्ष्य भी सम्मिलित होते हैं, जो इन लक्ष्यों की स्तर वार एवं कक्षा वार विभिन्नता एवं प्रथमिकता का निर्धारण करते हैं। इसलिए गणित का ज्ञान, शिक्षण के लिए गणित के ज्ञान से कई मायनों में अलग हैं। शिक्षकों को अवधारणाओं तथा प्रक्रियाओं को सही समझने एवं सही प्रस्तुतीकरण करने की आवश्यकता है साथ ही साथ यह भी आवश्याक है कि इन अवधारणाओं एवं प्रक्रियाओं के आधारों को भी समझें। इस प्रकार गणितीय संवेदनशीलता को भी एक अलग नज़रिये से देखा जाना चाहिए, गणितीय संवेदनशीलता कक्षा में बच्चों द्वारा किए गए गणितीय प्रयासों को समझना है, जिसमें बच्चों का ज्ञान, वे गणित कैसे सीखते हैं, विभिन्न समयान्तराल में अपने आसपास से सीखे गए गणित तथा सीखने की प्रक्रिया में प्रत्येक बच्चे का क्या होता है, को समझना है।

वर्तमान व्यवस्था के अन्तर्गत पुस्तकें तो अच्छी होनी ही चाहिए इसके साथ यह भी आवश्यक है कि शिक्षकों की तैयारी को भी परम्परागत प्रशिक्षण से हटकर सोचना होगा क्योंकि पिछले 20 साल के सेवाकालीन प्रशिक्षण से कोई बड़ा बदलाव नहीं दिखा न ही सेवापूर्व कार्यक्रम कोई बदलाव लाने में सक्षम हुए इसलिए अपनी क्षमता विकास की जिम्मेदारी शिक्षकों पर ही छोड़नी होगी प्रशिक्षण संस्थाओं की भूमिका संसाधान उपलब्ध कराने में सुविधादाता की होगी, प्रत्येक शिक्षक कों स्वयं निर्णय लेना होगा कि उन्हें किस क्षेत्र में अपनी क्षमता का विकास करना है तथा कैसे करना है!

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Study of Children's Errors: A Window to the Process of Teaching and Learning of Mathematics

Abstract

Errors that children make are often seen only as flaws in learning and rarely as windows to their thought process. Two ways of looking at errors include; viewing them as learning stages and viewing them as gaps in learning. These views become even more pronounced in the teaching-learning process of a subject like Mathematics where the binary of correct answer and incorrect answer is seen to be clearly distinct. In this paper a study of children's errors has been undertaken to understand what they indicate about the current knowledge of the learners. So, the new perspective being proposed in the paper is to view errors as an important resource for the teacher that would help them plan future teaching.

Introduction

Mistakes made by children elicit very different responses from adults based on the context in which they occur. The response and interaction is different if the mistake occurs among peers, in front of elders at home or in a formal setting like school. Usually at home and among family members, the common 'mistakes' of young children are taken very lightly and would elicit supportive, positive and affectionate responses. On the other hand in school, mistakes are generally perceived as undesirable, and are in the category of 'must be eradicated as soon as possible' form of behaviour, but we merely need to observe a child trying to learn something to realize that errors are an integral part of the learning process. This is true for all walks of life, whether academic or non-academic. Let us take a common example of a child learning to use a spoon for the first time. At first, she spills the food, but gradually, after a number of failures, succeeds. Similarly, the mistakes made by a child learning the names of the colours, which she may learn over a period of many months, reveal the systematic errors in the process of learning. At the first stage my two and a half year old daughter used the names of the colours merely as nouns, with little knowledge of what is green. She would point out to objects and say 'this is green'. Then she started using the words in the specific context of colours,, but did not know which colour was what and the third stage which is her current stage, she recognises black, but mixes up the names of all the other colours. Though she is able to identify two things of same colour and take notice of their common characteristic. At each stage of learning about colours, she makes diferent mistakes and as parents we are only focused on what she knows, happily ignoring what she doesn't know.

This paper is based on a study to understand the mistakes made by students and what they reveal about the current conceptual understanding. Place value, a fundamental concept of Mathematics curriculum of elementary classes, was taken as the area of enquiry to understand the nature of errors made by students and the reasons for them.

In India, the board to which the school is affiliated to, impacts education in many ways. The specifics of the syllabus, the textbooks prescribed and the assessment procedure depend on the board. Thus, to ensure variety in the

data, the sample included children from different settings: two Central Board of Secondary Education (CBSE) schools in Delhi, one CBSE school in Rajasthan, one state board school in Delhi and one state board school in Rajasthan.

Understanding Errors

When we examine the work done by children there are many ways in which we respond to it. When a two three year old child, drawing on a paper with a crayon says I am drawing a grape and then draws a big circle two types of responses are possible, "Oh great this is such a nice grape" without bothering with the lack of likeness. Another response could be, "this is good, but don't you think this is too big for a grape?" These responses indicate two very distinct and in a sense opposite ways of understanding errors. These two ways are:

- **Errors as learning stages:** The errors are a part of the learning process and apart from oversights that the child would not repeat, there are certain errors that would be made by almost all children going through that process. These are unavoidable as these are stages of learning. For example, all children pass through the following stages when learning to speak: Crying, Babbling, One Word/ Holophrases, Two Word Phrases and then Multi-Word Phrases (Aitchison, 1998). Whatever attempts a parent may make to avoid these in their children, these stages are inevitable and are generally not affected by correction except when the favourable time arises.

- **Errors as gaps in learning:** This would imply that children make errors because they have not been told the correct method or have not grasped the correct facts. The child tries to fill these gaps in her own logical, but not necessarily correct way which leads to errors. For example, if a child is shown a rectangle divided into five unequal parts as shown below and she answers that the fraction of the shaded region is one-fifth then the child is unaware or currently ignoring the fact that the five parts have to be equal for it to be one-fifth.

It can be seen that these two views are fundamentally different from each other. In the first view, errors are seen as necessary stages in the process of learning and these reflect the way a child thinks. They are evidences of children's knowledge rather than of their ignorance. And in the second approach they are the evidence of lack of knowledge.

Error analysis helps teachers in understanding what an error reflects about the child's current knowledge status. That is, it allows teachers to diagnose the level of learning of the students. This understanding helps her in modifying her approach to suit the needs of the children. Wrong answers given by students often tell us more about their present and unique state of understanding as compared to their right answers. There is often only one correct answer, but a variety of incorrect ones. Therefore, why and how a child reached a different answer becomes an interesting area of inquiry. Asking the student 'how' the question was solved (or 'speculating' about it based on the teacher's knowledge of what all the child already knows) would reveal a lot about the conceptual structure of that particular child's thought process. We would later talk more about these processes of understanding the errors made by children.

Context of Learning Maths

While learning Mathematics, children are supposed to learn abstract concepts and relationships as well as algorithms and facts (like number facts and

multiplication tables). There are three basic elements of any mathematical concept *(Richard Garlikov, The Concept and Teaching of Place-Value, 2000)*. The three elements are:

1. Convention
2. Algorithmic manipulations
3. Logical/conceptual relationships

However, due to over emphasis on learning and teaching of algorithms no time is given to building logical relationships. Teachers have themselves studied through this culture where convention and algorithm is all important. Thus as teachers they by default emphasise the same.

Therefore, children also either fail to grasp the concepts and principles that underlie procedures or they grasp relevant concepts and principles, but cannot **connect** them to the procedures. Either way, children who lack complete conceptual understanding, frequently generate systematic patterns of errors (Siegler, R.S. 2003). Siegler talks of how, depending on the manner in which one looks at it, these systematic errors can either be a problem or an opportunity. They are a problem as they indicate that children do not know what we have tried to teach them. On the other hand, they are an opportunity as they indicate the specific misunderstanding developed by a learner and thus can be worked upon.

Place Value in Mathematics: The way we normally record numbers is known as a decimal 'place value' system (Dickson, et.al. 1984). It is a system of successive groupings where units are grouped into tens; tens are grouped into hundreds, hundreds into thousands and so on.

Place value is one area with which the students start working (intuitively) at a very early stage. This means that they pick up the pattern in which numbers are generated or some understanding that 21 will be followed by 22 and 23 similarly 31 will be followed by 32 and 33. This develeops as children orally learn the number names and start learning to write the number names. When children start writing numbers they pick the number pattern and are able to predict next numbers. The ability to predict numbers is an indicator that they have understood something about the number structure. This is supported by the nature of number names. In English number names, after twenty are regular and indicate their decomposition i.e. Fifty one is fifty and one. (Nunes and Bryant, 1996).

But this is also an area in which students often make mistakes. Place value forms the basis of arithmetic and is thus related to errors in various other topics. As a result, children often form incorrect procedures and inefficient strategies for solving multi-digit arithmetic problems.

Elementary school teachers generally understand enough about how to use 'place value' to teach most students to eventually be able to work with it; but they don't often understand place value sufficiently to help them understand it very well, conceptually and logically. And they may even unknowingly impede learning by confusing children; for example, trying to make arbitrary conventions or giving recipes and short cuts as logical steps. In many primary schools, children chant one one eleven, one two twelve.....and so on, or they write in their notebooks following the two steps given below.

1st Step	2nd Step
1	11
1	12
1	13
1	14
1	15
1	16
1	17
1	18
1	19
2	20

This way of speaking and writing hinders the understanding of number sense.

Stages of development of place value: *As* discussed above learning is an ongoing process. One keeps adding more nuances to the understanding of a particular concept and build more relationships among the concepts. Understanding of place value similarly develops gradually. Ginsburg identifies three stages in developing an understanding of the theory of place value, where the written symbolization of the number is concerned.

- The first stage is where the child writes a number correctly with no idea as to why it is written in this manner. For example thirteen is written as '13' and there is no reason for it.
- The second stage is where the child realizes that other ways of writing a particular number are wrong - for example '31' is incorrect for 'thirteen.'
- In the third stage the child is able to relate the written notation of numbers to the understanding of place value. For example, Doug, a 7 year old, when asked why he had written a '1' followed by a '3' to indicate 'thirteen' replied that the '1' stands for ten and '3' stands for 3. Ten and three is thirteen.

Thus even though ch1ildren in stage 1 and 2, start unearthing patterns and develop an understanding that numbers proceed in a systematic manner, they are unable to articulate that system. Thus, the shaky concepts that they have, do not support in forming effective arithmetic strategies. It would thus be worthwhile for a teacher to understand the depth of learning of the children to take them to the next level.

Sample selection

The sample of the study included children studying in class 7 from five different schools. These schools catered to populations from varied socio-economic background and using different Mathematics textbooks. Three schools used NCERT books, one school used the Delhi state board textbooks and one school had Rajasthan state board textbooks.

Place value is formally introduced in the textbooks of class 3. It is a fundamental concept in Mathematics and later number system, arithmetic and algorithms develop on it. The spiralling of curriculum and working with and on the concept of place value provides students with an opportunity to understand the concpt better. Thus it was decided to undertake the study with class 7th students and the problems for the study were selected from the textbooks of class 6th.

All the students of class 7 were given a test paper. Based on the errors made by students in the paper, three students in each school were selected and follow up work done with them. They were asked to do a few of the questions again along with unstructured interview. While selecting questions to be given to each child in the follow up session, it was considered that the first question to be given to them would be the one which the child had correctly answered in the test paper. Four more questions were given which the child had answered incorrectly. Numbers in the questions given in the follow up were changed. The interview focused on asking the children what they understood by the question, how they think it needed to be solved and then allowing them to solve it while encouraging them to articulate the reason for what they were doing. The interviewer recorded both the strategy undertaken by the child and her articulation of the question and required solution.

Problems related to number sense

Based on the responses in the paper and in the interview, errors have been categorised as follows:

1. Errors originating from difficulty in comprehending mathematical language
2. Errors originating from the understanding of place value

Comprehension of mathematical language: Mathematical language or the language used to communicate mathematical ideas has many peculiar features to it like use of symbols, words having meaning different to the customary ones, use of charts, tables and graphs, language of word problems, use of conditions, etc. This causes an additional challenge for the learners of Mathematics. Like the learning of any other language, the learning of mathematical language is impacted by the amount of exposure and the usage of language to communicate ideas and not focus on the language itself.

Errors originating from difficulty in comprehending mathematical language: There were a large number of mistakes in the paper that were related to the understanding of mathematical language. These mistakes related to use of symbols, understanding the language of the question and difficulty in dealing with questions that had more than one condition.

In the following paragraphs, a brief description of such errors has been presented:

Confusion about symbols - what should be written where, knowing what a symbol stood for, respecting what it means and how we should relate it to the question in view of it was missing in many students. There were several examples where students incorrectly and interchangeably used the comma or the equality sign. For example while writing the number 9857 a child had written 9,8,5,7. In another place an equality sign was used to separate different numbers (7744 = 7474 = 4477 = 4747).

Another error in the use of the comma was that many children in the entire test, did not place the comma even at the places it was needed. Thus to distinguish one number from another space was left. *For example when asked to write the greatest and smallest 4 digit number using all different digits, the child wrote 9876 1023, with no comma to separate the numbers.*

Errors due to language confusion: Another problem which one often faces while trying to assess a child's mathematical learning is how to differentiate between whether the child is unable to capture the concept or is confused with the vocabulary/language.

The first question asked children to write the greatest and the smallest number from a given set of numbers. While answering it, three students wrote the given numbers in ascending and descending orders instead of choosing the greatest and the smallest number. Nine students picked the two greater numbers and wrote them both instead of the one greatest number. When a child was asked what he understood from the question, he said, "We have to set these numbers according to the greatest and the smallest, first we will write the greatest number, then smaller than that and then even smaller. Greatest means from bigger to smaller."

Similarly in the question where certain number names were to be written as numerals, many students wrote the number correctly using Roman numerals. This could be because of the interference of the word 'numeral' that they had encountered in the context of Roman numerals.

Commonly used vocabulary also posed problems for the students in some cases. Some words, when used in

mathematical context have a different meaning than in their everyday usage. These include words such as point, equality, chance, etc. One such word, used in the test, was difference. In the question where the students were asked the difference between the two place values of 2 (Q9), 27 students took the word difference in the literal sense. Thus they gave some very interesting answers like 20000 is greater than 20 or it has more number of digits, it has 4 zeroes, etc.

In Q10, where six digits 4, 5, 6, 7, 0 and 8 were given and students were asked to make 5 six-digit numbers and then arrange them in ascending order, some students instead of placing the numbers in ascending order placed them in descending order thus showing that while aware of the ordering of the numbers they are still confused about the nomenclature.

Sometimes when part of the question is done incorrectly and we try to identify the reason for it, we may find that the problem was not with the concept, but incorrect or partial comprehension of the question. In such a case rephrasing the question may help them to find the correct answer. In a question where 4 digits 2, 4, 7 and 8 were given and students were asked whether the smallest 4 digit number (made using the given digits) would be greater than 3000, as many as 30 students wrote the answer 4278. A probable reason for this could be that students misunderstood the question and interpreted it as asking them to write the smallest number which is greater than 3000. Some other answers like 3001 and 3002 also support this understanding.

In lengthy questions, chances of misinterpretations are even higher. In a question "you have the following digits 4, 5, 6, 7, 0 and 8. Using these, make five different numbers with six digits each. Now arrange these numbers in ascending order." 12 students did not make the numbers using the given digits; instead they arranged the digits themselves in ascending order.

In Q3, where the students are asked to expand the question, some students wrote in tenth and hundredth. This is something that these students have started learning in class 7 and thus this probably is interfering with the concepts learnt earlier. In the question asking them to give place value of digits placed at different positions in the numeral for a number, some students said that the place value of 2 remains 2 in any positions. Face value is introduced as a term to students much later than place value; thus a more recently learnt concept is fresh in their minds and also interferes with the earlier concepts. We can say that they are using the more recently learnt concepts and that is because of two reasons. One is linked to the fact that since they are in the process of acquiring these concepts they end up attempting to link it to everything they come across to explore and test if it can be related or not. The second reason is because of the way Mathematics and other subjects are taught where once an idea is introduced the classroom works with those ideas for many days continuously till the next idea comes. So they are all expecting to be given questions related to recently learnt concepts only and hence the wrong interpretations.

In many cases, it was felt that the students could not focus on all the conditions given in a question and thus simplified or solved part of the question. This was a pattern seen across the schools.

In the question where the students had to make the smallest and the greatest 4-digit number using digits that are all different, they simplified the question for themselves by only focusing on the condition of making the smallest

and the greatest number. Thus we got to see answers like 1000, 1111 and 9999.

In the other question the task was similar, but there was a slight difference. Here they had to make the smallest 4-digit number while using the given three digits only. In this they were required to use one of the digits twice and could choose whichever they wanted, but most of the students could not perhaps understand and did not comply with the conditions. Many made 4-digit number using any random digit other than the 3 that were given. Some other students used all the given digits and made the smallest 3-digit number possible. Another child gave 1000 as the answer thus focusing only on that part of the question which said the smallest 4-digit number.

Errors related to place value

- ***Using Zero***: Zero has been a problem spot for many children and the inability of students to work with numbers containing the digit zero was seen in many questions. In the question where students had to expand the number 20085 (Q3), many students gave the answer 20000+0000+000+80+5 or 20000+0+0+80+5.

  Thus the students either do not know or are not very confidently aware of the fact that zero is only a place holder and thus has no value that needs to be separately written. Leeb Lundberg (1977) describes some of her problems as a teacher when dealing with zero. Its role as a placeholder, in the symbolic representation of number, is something not readily appreciated by children.

- When asked to write ten thousand and nineteen and thirty three thousand and thirty three as numerals a girl wrote them as 1019 and 3333. When she was asked to explain, she said, "We were asked to write ten thousand and nineteen in numerals, so we write 10 and since zero has no value, so we write 19 after that. Similarly we write 33 and then 33. If we write 3300033 then it will be entirely wrong." (The response has been translated)

  As we see here students were expected to write the given number names in numeral form, the specific requirement of the question was to use zero as a place holder. Most of the errors that were seen in this question showed that students find it difficult to place zero as a place holder and even when placing zeroes are not sure how many zeroes are exactly needed. Responses like 1000019 and 3300033 are interesting as in these first ten then three zeroes are written for thousand and then nineteen is added. The response is similar in the second case.

- ***Difficulty in working with large numbers:*** There were many examples when it was felt that the problem that students faced was not in understanding of the concept, but in handling large numbers. This is an area where even secondary school children show a definite weakness. Many seem to be unfamiliar with the place names of digits to the left of thousands position (Dickson, L. et.al; 1984). This could also be one possible reason for such a large number of students finding it difficult to write ten thousand and nineteen and thirty three thousand and thirty three. Answers which had many more zeroes than needed could also be because of this. Their competence in dealing with large numbers may not have yet developed enough to check the numeral they have written and what number it actually

represents. There were other places where errors indicated difficulty in handling large numbers.

While writing 20085 in expanded form, many decreased or increased number of zeroes, but read out the number correctly. This suggests that if the number of digits are large, students are not able to make sense of the number and get confused. This is what perhaps makes them give wrong answers.

Similarly in the question where the students had to write numerals for given number names, there were several incorrect attempts and again the most frequent mistake was writing a 4 digit number instead of 5 digits.

Many students wrote 2000 for the place value of 2 in 426328. This also seems to indicate that they find the number represented by the numeral too big to handle. It is possible that it is only a slip, but the frequency of this error makes this being only a chance error or a slip a little improbable.

Large numbers seemed to be creating a problem even while carrying out familiar algorithm operations. The operations needed in this test were subtraction, multiplication and division. Though children faced difficulties in all the operations, it was more so in the case of division. While working on division questions, 11 students started working on the algorithm correctly, but left it midway, even though many of them had done it correctly till that point. Thus not knowing how to do the sum was not the reason for their leaving the question; instead it seems that the length of the question scared or bored them. Some kept on doing it, but forgot to write the quotient.

- ***Basic algorithms*:** Many students in spite of being aware of the demands of the question and being aware of the concepts end up making mistakes in carrying out simple operations. Many such cases were seen in the test also. Students who could give the correct place values and had also interpreted the word "difference" correctly, i.e. as per the question made mistakes while subtracting.

  Eleven students while multiplying 1098 with 25, made an error in vertical addition. Eight students forgot to add the carry over. Similarly while working on division questions many students did the subtraction part wrong. Most of the children who make such a mistake when trying to spot it on their own, were able to find it.

  Question 11 required students to subtract 6980 from 10000. In this question the most frequent incorrect answers were 4120 and 2120. Children borrowed from 10 repeatedly, thus 10 minus 8 gives 2 and 10 minus 9 gives 1. From the first 10 either they subtracted 6 to get 4 or considered it to be 8 as twice they have taken one from it thus 8 minus 6 is 2. This indicates how the number notation, relationship between different places and the related idea of borrowing is not clear to them. It also raises questions whether borrowing is a correct idea to be presented with the subtraction algorithm.

Conclusion

The analysis shows the different areas of difficulty students face in working on place value related problems. The challenges include the use of zero as place holder and the algorithm of carry over, borrowing, multiplication and division originating from the

understanding of value attached with the place of a digit in a base ten number system.

This small indepth study of the understanding of place value is indicative of the variations and layers that can be seen in the errors made by children relating to a small curricular area in Mathematics namely place value. This has many implications for the teaching/learning process in elementary classes.

The first and perhaps one of the most important implication is about how we view the errors made by children. Instead of the popular view of errors as a sinful deviation which needs instant reprimand and correction, it is possible to view them as a window to the understanding of children. A teacher can form 'reasoned speculations' based on her knowledge of child's learning level and her own conceptual clarity. Collaborative engagement of the learner and the teacher on these errors may lead to learner's development towards correct understanding on her own.

One on one interview often used by researchers to gain deeper understanding of all kinds of issues in various disciplines, is also an important pedagogic and assessment tool. The traditional paper pencil test can only tell us about the question that the child could do correctly and the ones that she could not. One on one interaction on the other hand helps teacher understand the level of understanding as development of a concept is a graded process and not a binary of knowing and not knowing. Asking the child, what is it she understands by the question and what does she think is needed to be done are important mathematical learnings even when she is unable to work the algorithm and makes mistakes in it. The second step is to understand how she would work the algorithm and if she can articulate why this is being done. This articulation on one hand would help the teacher, but on the other hand, it would provide an invaluable opportunity to the learner to reflect on what she is doing, gain confidence about it and understand that Mathematics is not a bunch of baseless algorithms, but an intricate network of concepts.

The question of how much does a teacher need to know to be able to teach is a well debated question. It is linked to the mandatory qualifications required for teachers and the content of the in service programme. This study throws some light on this aspect. Considering the real understanding of concepts in Mathematics has many layers to them, a teacher is on one hand required to know the concept fully and also be aware of the stages in development of the concept. The knowledge of how a concept develops is essential for designing both pedagogic and assessment activities.

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गणित और पाठ्य-पुस्तक

सार

हमारी शिक्षा प्रणाली का महत्वपूर्ण हिस्सा है- पाठ्यपुस्तकें जिन्हें पढ़कर विद्यार्थियों को परीक्षा पास करनी है। यदि पाठ्य-पुस्तकें बेहद रूचिकर हो, विशेष कर प्राथमिक कक्षाओं की तो बच्चों का पढ़ाई से लगाव बना रहता है। परन्तु यदि ये पुस्तके बेहद ऊबाऊ और उत्तर केन्द्रित हो विशेष कर गणित में तो वे बच्चों को हमेशा के लिए गणित से दूर कर देती है। क्या गणित की प्राथमिक स्तर की पाठ्य-पुस्तके रूचिकर, सन्दर्भ से जोड़ने वाली और बच्चों को जूझने के मौके देने वाली नहीं होनी चाहिए।

विद्यालयों में गणित पढ़ना और पढ़ाना दोनों का तात्पर्य है गणित की पाठ्य-पुस्तकों में दिए गए सवालों के उत्तर प्राप्त करना। इन प्रश्नों के उत्तर के लिए या उन्हें हल करने के लिए विद्यार्थी पूरी तरह से शिक्षक पर निर्भर रहते हैं। शिक्षक भी इस ओर कम ही ध्यान दे पाते हैं कि विद्यार्थियों की इन प्रश्नों के बारे में क्या समझ बनती है या वे प्रश्नों को हल करने के लिए कितना प्रयास करते हैं।

अधिकांश शिक्षक जो गणित पढ़ाते है उनकी तैयारी भी ऐसी होती है कि वे सवालों के हल बच्चों को दे देते है और उनका अभ्यास बार-बार करवा कर उन्हें परीक्षा में अच्छे अंक प्राप्त करने के लिए तैयार कर देते हैं। इस प्रकार की तैयारी में कक्षा के अधिकांश बच्चे गणित में पीछे छूट जाते हैं और आगे चलकर गणित के प्रति उनका रूझान खत्म हो जाता है।

मैंने भी एक शिक्षक की हैसियत से जब प्राथमिक कक्षाओं में गणित पढ़ाना प्रारम्भ किया तो पाठ्य-पुस्तकों के सवालों को हल करवाना ही मेरा भी उद्देश्य होता था। जब मैंने जाना कि इन सवालों को कुछ समय बाद फिर से हल करने को कहा जाए तो बच्चे उन्हे नहीं कर पाते हैं यानि उनके दिमाग में सवाल ने कोई छाप नहीं छोड़ी। मुझे लगा कि मैं तो सारा समय व्यर्थ ही बर्बाद कर रही हूँ। बच्चों में सिखाई हुई बात नज़र नहीं आ रही है, वे तो वहीं के वहीं हैं। तब मैंने बच्चों के साथ गणित से सम्बन्धित वार्तालाप की शुरूआत की। परन्तु उससे भी विशेष कुछ नहीं हुआ। विद्या भवन, उदयपुर में कार्य करते हुए मुझे कुछ ही वर्ष हुए थे कि मुझे गणित की कार्यशाला में जाने का मौका मिला। वहाँ गणित पढ़ाने की आवश्यकता और गणित विषय विशेषताओं को जानने व समझने का मुझे सुअवसर प्राप्त हुआ। कार्यशालाओं में मैंने जाना कि बच्चों की गणित पर समझ बनाने के लिए worksheet की मदद ली जा सकती है। इसके बाद मैंने अलग-अलग अवधारणाओं के लिए worksheet बनाना शुरू किया और समय-समय पर बच्चों को worksheet देकर उनकी समझ को जाँचने-परखने का प्रयास किया जिससे उनकी समस्याओं को दूर करने में मुझे मदद मिली।

विद्यार्थी worksheet से माथापच्ची तो लगातार करते हैं, परन्तु अवधारणाओं की समझ स्पष्ट न होने से बच्चे सवालों के साथ प्रयास तो करते हैं लेकिन उनकी समझ ज्यादा समय तक नहीं रहती इसलिए वे दोबारा वैसे ही सवालों को जब हल करते हैं तो उन्हे फिर नयी सोच से काम करना शुरू करना पड़ता है। यानि पहले किए गए काम की समझ उनको आगे के कार्य के लिए कोई आधार नहीं देती। बच्चों के इस कार्य से मुझे महसूस होने लगा कि बच्चों की गणितीय समझ बनाए रखने के लिए कुछ गतिविधियों की आवश्यकता है, जो उनके दैनिक कार्यों से जुड़ी हो।

यहाँ मैं कक्षा III से VI तक के विद्यार्थियों के साथ कक्षा में की गई गतिविधियों के बारे में बताना चाहती हूँ। जिससे बच्चों को कई अवधारणाओं को समझने में मदद मिलती है। कक्षा-IV के बच्चे मीटर-सेमी को बदलते हुए कई तरह की गलतियाँ करते हैं और उसका अंदाजा भी नही लगा पाते। जब बच्चों से मैंने कहा कि हमारे पास

लम्बाई नापने के लिए मीटर स्केल नहीं है तो हम क्या कर सकते हैं? कैसे अपना स्केल बना सकते हैं। बच्चे एक लम्बी लकड़ी ले आए कि हम इससे एक मीटरस्केल बना सकते हैं, उन्होंने 15-15 सेमी नाप कर पूरे 100 सेमी तक निशान लगाकर स्केल बनाया और फिर बरामदा, कमरा आदि स्थानों की लम्बाई नापी और मीटर-सेमी में लिखी। इसी तरह उन्होने 1-1 वर्ग मीटर के निशान बनाकर कमरे की लम्बाई-चौड़ाई से क्षेत्रफल निकालना समझ लिया।

कक्षा III-IV के बच्चे रूपये-पैसो का हिसाब नहीं समझ पाते, उन्हें नकली नोट से कक्षा में दुकान लगाकर ही लेन-देन करना बताया गया। बच्चों ने दुकान के खेल में अपने-अपने सामान की सूची बनाना उसके पैसे लिखना और खरीदे गए सामान का हिसाब लगाना आसानी से सीख लिया और फिर उसे सवालों के रूप में पढ़कर हल करना उनके लिए सरल हो गया।

मुझे मेरी कक्षा में बच्चों के साथ काम करने की पूरी आजादी थी इसलिए मैं ऐसी कई गतिविधियाँ उनके साथ करने लगी। इस तरह कार्य करते हुए बच्चों की भी गणित के प्रति रूचि बढ़ने लगी। परन्तु अगली कुछ कार्य-शालाओं में मुझे लगा कि सभी शिक्षक ऐसा नहीं कर पाते इसलिए पाठ्य-पुस्तको में ही कक्षा I से V के लिए किसी भी सन्दर्भ से अवधारणओं को जोड़ते हुए समझाया जाना चाहिए ताकि बच्चों को गणित जीवन से जुड़ा महसूस हो। ऐसा एक प्रयास (2013 में) राजस्थान की पाठ्य-पुस्तकों में बहुत सोच समझ के साथ किया भी गया और वह काफी सफल भी रहा जिसे शिक्षकों ने भी सराहा। इन पाठ्य पुस्तकों में किये गए एक अन्य प्रयास को भी शिक्षकों ने सराहा। इन पुस्तकों में गतिविधियाँ करने के साथ-साथ एक महत्वपूर्ण कार्य जो शिक्षकों को करना था और कईयों ने बहुत शौक से किया वो था नये सवाल बनाना। इस पाठ्य-पुस्तक की कुछ गतिविधियों की भी मैं चर्चा करना चाहूंगी जिससे बच्चों को गणित सीखने में बहुत मदद मिली।

कक्षा II व III में मोतीमाला के द्वारा पाठ्य-पुस्तक में गिनना व जोड़-घटाव का काम बच्चों को इस तरह से करने का मौका दिया गया कि वे संख्या की समझ भी बना सकें और उसको बढ़ते घटते हुए भी देख सकें। बच्चे सामान्यत: खड़ी लिखी संख्याओं को ही जोड़ सकते हैं। उन्हें 17+19 लिखकर जोड़ने के लिए दिया जाए तो वे संख्या को खड़ी लिखने का प्रयास करते हैं। परन्तु मोतीमाला पर वे इस जोड़ को मोती गिनकर लिख सकते हैं। और धीरे-धीरे अभ्यास करने पर वे बड़ी संख्या को भी मोतीमाला पर पहचान कर संक्रिया (जोड़-घटाव) कर लेते हैं। इसके बाद बच्चे अपने स्वंय सवाल बना लेते हैं।

इससे आगे का एक उदाहरण है संख्या को संख्या रेखा पर दर्शाना। कक्षा-IV की पुस्तक में गाँव के मुख्य स्थानों को रेखा पर दर्शाया गया और दूरियों पर सवाल बनाए गए थे। अतः बच्चों ने भी अपने गाँव के मुख्य स्थानों को संख्या रेखा पर लिखकर नए सवाल बनाए और आपस में मिलकर हल किए।

कक्षा-V के बच्चों ने दो साल में जो समझ बनाई उसके आधार पर उन्होंने हिसाब-किताब समझते हुए अपनी कक्षा में बोर्ड पर भाव सूची लिखी और अपने-अपने सामान का बिल बनाया। यहाँ हर बच्चे के पास अलग बिल था और उसका अपना अलग सवाल था।

कठिन अवधारणाओं को भी ठोस वस्तुओं के साथ और खेल-खेल में बच्चों ने आसानी से सीख लिया। पाठ्य-पुस्तक में सिखाए गए बीज जमाने के तरीके से बच्चों ने छोटी-छोटी संख्या के गुणनखण्ड के खेल में उन्होंने वे संख्या भी समझ ली जिसके गुणनखण्ड केवल 1 या स्वंय संख्या ही है। जैसे:- उन्होंने दूसरी टीम को 13 या 17 संख्या दी जिसके अधिक गुणनखण्ड दूसरी टीम को न मिल सके।

इस प्रकार गणित की आकृति, स्थान और मापन जैसी अवधारणाओं पर बच्चों ने कई छोटे-छोटे प्रश्न बनाने की सफल कोशिश की जिसका फायदा शिक्षको और बच्चों को लम्बे समय तक होता। परन्तु किन्हीं कारणों से या मूल्यांकन की परेशानियों से सरकार द्वारा 2016 में ही पाठ्य-पुस्तकों को बदल दिया गया। और एक बेहद अच्छा प्रयास जो लीक से हटकर शिक्षक को स्वयं प्रश्न बनाने और सोचने के लिए तैयार कर रहा था वह बन्द हो गया और उसका स्थान फिर से ''प्रश्नावली'' ने ले लिया। जिसने शिक्षकों की गणितीय सोच पर फिर से ताले लगा दिए।

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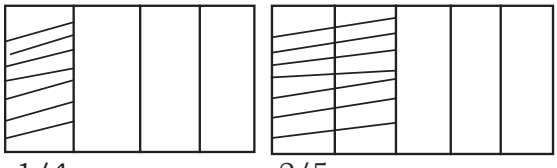
भिन्नात्मक संख्या को समझे या रटें

सार

कहते है गणित हमें तार्किक बनाता है, परन्तु इसके लिए हमें भी गणित को कई तरीको से जानने की कोशिश करनी पड़ती है। ऐसी ही एक कोशिश मैंने की खोजबीन से यह जानने की कि भिन्न के लिए हम किसी ईकाई में बराबर हिस्से ही क्यों करते हैं? इस माथा पच्ची से बहुत कुछ नया सीखने को मिला।

पाठ्यपुस्तकें, शिक्षक प्रशिक्षक और हम शिक्षक भिन्न को कुछ इस तरह समझाते हैं

1. किसी ईकाई के हिस्से को भिन्नात्मक संख्या से दिखाते हैं
2. इसके लिये एक आयाताकार चित्र को एक से अधिक बराबर हिस्से मे बाँटकर उसमे से एक या अधिक भाग को छायांकित किया जाता है। फिर छायांकित भाग वाले हिस्सों की संख्या बटे(÷) कुल हिस्सों की संख्या को भिन्न लिखने के कायदे के रुप मे बता दिया जाता है।



इस तरह के कुछ चित्रों के अभ्यास के बाद सीखने वाला चित्र में बनी आकृति को भिन्न की शब्दावली मे पढ़ने लगता है। आगे इसी तरह के पहचान आधारित नियमो को जानकर वह सम भिन्न, विषम भिन्न, तुल्य भिन्न, बड़ी छोटी भिन्नों की पहचान और भिन्नों पर संक्रियायें करना भी जान जाता है। पर क्या यह पहचान आधारित जानकारी गणित या भिन्न की समझ का कोई गहरा मौका उपलब्ध कराती है?

मेरी कक्षा में मैने बच्चों को इस तरह चित्रों से भिन्न समझाकर फिर कागज मोड़कर 1/5 दिखाया। अब मैने बच्चों को कहा वे कागज मोड़कर मुझे 1/4 हिस्सा दिखाएं। कुछ देर बाद एक लड़की ने मुझे अपना कागज दिखाया। उसके मोड़े हुए हिस्से बराबर नहीं थे। मेरे यह कहने पर कि उसने हिस्से बराबर नहीं किए हैं उसने मुझसे पूछा, ''हिस्से बराबर क्यों होने चाहिए? ''

मेरे पास तुरन्त कोई जबाव नहीं था वैसे मैं यह कह कर टाल सकता था कि 1/4 का मतलब है, किसी ईकाई के चार बराबर हिस्सों मे से एक हिस्सा लिया जाना। पर उस लड़की ने यही तो सवाल किया था कि बराबर हिस्से क्यों? यह सवाल नियम 'क्या है' का न होकर नियम 'क्यों है' का था। मैनें भी इस बारे में पहले नहीं सोचा था। पर अब लगा इसे समझना चाहिए।

मैंने खुद इसे समझने के लिए एक कागज लेकर उसके चार टुकड़े किए जो बराबर नहीं थे। इनमें से एक टुकड़े को उठाकर मैनें खुद से सवाल किया, ''क्या यह हिस्सा 1/4 है?''। यदि 'हाँ' तो क्यों और 'नहीं' तो क्यों नहीं।

मेरा उत्तर नहीं था, क्योकि यदि यह टुकड़ा 1/4 होगा तो दूसरा टुकड़ा भी 1/4 होगा, तीसरा और चौथा भी1/4 होगा। इस ईकाई के सारे 1/4 एक जैसे साइज के होने चाहिए जबकि ऐसा नहीं है। पर तभी मुझे दूसरा प्रतिकारी तर्क सूझा कि पहला टुकड़ा तो 1/4 हो सकता है, पर बाकी के तीन टुकड़े अलग अलग भिन्न के हैं क्योंकि ये एक समान आकार के नही हैं। चूंकि चारों टुकड़े जोड़ने पर एक ईकाई या पूरा कागज बन जाता है। जैसे 1/4 +1/5 +1/3 +13/60 = 60/60 = 1

मुझे यह तर्क अच्छा लगा पहले जबाव में मुझे लगा नियम हावी हो रहा था। खैर यदि मैं मान लूँ कि कागज का यह पहला टुकड़ा 1/4 हो सकता है और बाकी तीन टुकड़े अलग अलग भिन्न के तो मैं कैसे जानूँ कि यह पहला टुकड़ा 1/4 ही है?

यह सवाल बदलकर कुछ इस तरह का हो सकता है - यदि किसी ईकाई में से हम एक टुकड़ा निकाल लें और अब हमें यह तय करना है कि ये टुकड़ा कितना बड़ा है। (दैनिक समस्याओं में तो भिन्न इसी रूप में आती हैं) मैंने एक दूसरा कागज लिया ओर उसमे से एक टुकड़ा फाड़ लिया। मुझे कागज के इस हिस्से का मान पता करना था।

कहते हैं गणित हमें तार्किक बनाता है, जब हम किसी आकार या मात्रा की माप या गणना करते हैं तो किसी निश्चित ईकाई से उस आकार या मात्रा की तुलना करते हैं। यह मापक ईकाई उसमें कितनी बार आई है। यह तुलना ही मात्रा की गणना का आधार है। जैसे 40 पेज़ की कापी में, पेज की ईकाई 40 बार आई है। उसमें 40 पेज हैं। यह अजीब लगे तो ऐसे भी कह सकते हैं कि 5 मीटर लम्बे कमरे में मीटर की इकाई लम्बाई में 5 बार आएगी। इसी तर्क का इस्तेमाल हमारे इस कागज के टुकड़े को नाम देने के लिए किया जा सकता है।

इस टुकड़े की तुलना उस पूरे कागज से करनी होगी, जिसका यह हिस्सा है। इसके लिए मैं इस टुकड़े को कागज पर जमाता हूँ, और इसके आकार को पेन्सिल से चिन्हित करता जाता हूँ। इस तरह टुकड़े से पूरे पेज़ को नापता हूँ। इस तरह जमाने पर यह टुकड़ा 5 बार पूरा जमा 6वीं नाप में पेज से बाहर निकल रहा था। यदि यह टुकड़ा 6 बार पूरा पूरा कागज पर जम जाता तो हम कहते पूरा कागज इस टुकड़े से 6 गुना बड़ा है या यह टुकड़ा पूरे कागज का छटवाँ हिस्सा है और इसे 1/6 लिखेंगे।

परन्तु यह टुकड़ा कागज पर पूरा-पूरा नहीं जमा, यानी 5 बार जमाने पर कागज बच गया और 6 बार में टुकड़े का हिस्सा कागज से बाहर निकल गया। अब मैंने फिर तर्क का सहारा लिया। इस टुकड़े को छोटा करना होगा! मैंने टुकड़े के दो बराबर हिस्से किये और इन छोटे हिस्सों को पूरे कागज पर जमाया। यह हिस्सा 13 बार जमाने पर पूरा पूरा जम गया। टुकड़े का आधा हिस्सा 13 बार पूरे कागज पर जमा। कागज के 13 भागों मे से दो भाग हमारे पास हैं इसे हम 2/13 लिखेगें। अब मैं समझा कि सभी टुकड़े बराबर नहीं होंगे तो हम यह नहीं कह सकेंगे कि लिया गया टुकड़ा कितना हिस्सा है। अत: सरल यही है कि टुकड़े बराबर रखे जाएं।

यह करने के बाद मैंने बच्चों से बात की। मैंने हर बच्चे को एक एक चौकोर कागज दिया और उनसे पूछा कि आपके पास कितने कागज हैं सबने कहा एक है।

उसमें से एक टुकड़ा फाड़ने को कहा। अब उनसे पूछा कि आपने जो टुकड़ा फाड़ा है वह कितना है? बच्चो ने जबाव नहीं दिया। फिर मैंने कहा कि क्या ये टुकड़ा एक है? कुछ ने कहा हाँ, कुछ ने कहा नहीं। मैंने दोहराया कि एक तो हमने पूरे कागज को कहा था क्या ये पूरा कागज है? सभी ने एक साथ कहा नहीं! एक से छोटा है।

फिर मैंने अलग अलग उदाहरणों से तुलना करने की चर्चा की और टुकड़े की तुलना पूरे कागज से करायी।

इस तरह हम किसी ईकाई के हिस्से को मात्रात्मक मान देने के अनुभव से गुजरे। मुझे लगा कि यह पहले से बराबर किए हिस्से में से हिस्सों को लेकर भिन्न दिखाने से अधिक तार्किक था।

मेरा यह मानना है कि ऐसी खोजबीन से मेरी भी बेहतर समझ बनी और बच्चों ने भी कुछ नया सीखा। हो सकता है यह उलझा हुआ तरीका हो किन्तु हम सब को इस माथा पच्ची से फायदा हुआ। मैं आप लोगों की भी प्रतिक्रिया इस अनुभव पर चाहता हूँ।

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My Experiences with Mathematics Education

Abstract

Most adults when asked about their experience with mathematics would say that they are scared of it. This article tries to look at some of the reasons why this is the case. It is based on some of the experiences I have had while working with children, teachers and teacher-students.

A few days ago, a very enthusiastic math educator asked me about my views about 'fear of mathematics', She wanted to study a classroom and understand the factors that create this 'fear of mathematics'. While talking to her, I remembered all those instances I have seen this fear of mathematics and started wondering if studying one particular teacher or one particular classroom can actually help us.

In this article I would like to recall some of these instances that I have seen and try to look at the question, "Is fear of mathematics a local issue?"

A few years ago, I got a chance to visit a DIET in Rajasthan. When the DIET faculty introduced me to the student-teachers, the student-teachers seemed enthusiastic. But as soon they were told that I would be doing some mathematics activities with them I could see their enthusiasm reduce. I started talking to them about my session and asked how many of them liked mathematics and only one hand was raised. Most of them said that they were scared of mathematics. These student-teachers were soon going to be primary school teachers and were expected to teach mathematics to children so I wondered, how do we expect a teacher who is scared of mathematics to teach children to enjoy mathematics? I also thought about the possible reason for this, As Deborah Ball writes in one of her articles in the context on teachers in the U.S. which is applicable in the Indian context too, [1] "*That the quality of mathematics teaching depends on teachers' knowledge of the subject should not be a surprise. ....Equally unsurprising is that many U.S. teachers lack sound mathematical understanding and skill. This is unsurprising because teachers — like all other adults in this country — are graduates of the system we seek to improve. Their own opportunities to learn mathematics have been uneven, and often inadequate, just like those of their non-teaching peers.*" While introducing my session, I told them that I would be talking to them about fractions and I saw fear on their faces. Their keenness to know about what I am going to do was further reduced. We started talking about fractions and I saw that their understanding of fractions was very minimal. Most of them thought that 1/3 was greater than 1/2.

We started our session. The idea was to introduce the equal share meaning of fractions and work through some examples based on the research. After a series of examples of equal sharing with unit fractions (where the number of rotis to be divided is 1), we moved on to compare some fractions. Now 1/3 was not anymore greater than 1/2 because it was clear that if a roti is divided equally between 3 children and another is divide equally between 2, the share of children in the latter would always be greater than the one where there are 3 children. Suddenly fractions like 19/17 and 17/15 started making sense and most students-teachers

could verbally compare these two fractions. "If a child gets 19/17, she will get 1 roti and 2/17th part of another. In the case of 17/15, each child will get 1 roti and 1/15th part of another roti. 1 roti divided between 17 children, the share would be lesser than that of 1 roti divided between 15 children." This method was not only easier but also faster than actually multiplying 17 and 17 and comparing it with 19 and 15. After this some students actually went on to compare 103/101 and 5/3. They did not find fractions scary anymore. These fractions were only shares of each child when rotis were divided between children.

This session was only for four hours but I could see that more such sessions would definitely help the student-teachers be less scared of mathematics. Though I would like to believe that my session helped the students understand fractions, the truth is that it was the context which helped the students deal with the topic of fractions better. Fractions were no longer rectangles with parts that were coloured, but were things/numbers they would deal with, play with and understand. The context of equal division of rotis was familiar unlike the rectangles and their equal parts.

When I talked to the student-teachers later, some of them said that they understood the fractions they did that day, but how do they do fractions in mathematics? This question baffled me and I was unable to understand the question at all.

I understood the question much later during an interview. There was a candidate who was asked to divide 4040 by 8. Without even batting an eyelid he said 55 and showed us how he got it. When asked if Rs. 4040 were divided between 8 people, how much would each get, he said that it would be more than 500. And added that "but in maths the answer is 55."

Why was it that this person trusted his flawed mathematical algorithm more than his own common sense? Why did he accept the difference in the answer between the answer he got using his basic understanding and the answer he got using mathematical algorithm without question? Is it because the way mathematics is taught in our schools is extremely disconnected from real life? Is it because there is no attempt to connect it to our lives or to even see if the answer is meaningful and sensible?

For example, I remember a constant debate some of us had during designing a state curriculum. While writing the part on measurement, some of the group members wanted to use the word **वस्तुमान** (mass) instead of **वजन** (weight). Long arguments followed about using familiar words with students especially while dealing with young children and **वस्तुमान** (mass) won over **वजन** (weight). Moreover some of my sensitive physicist friends also supported this argument of using mass instead of weight as this can lead to misconceptions while doing higher physics.

A lot of this people who supported using the **'वस्तुमान'** (mass) instead of **वजन** (weight) ignored the fact that this was meant for 7 year old children who might not have ever heard the word **'वस्तुमान'**.

Due to our obsession with preparing students for higher studies are we risking reducing their interest in their current studies?

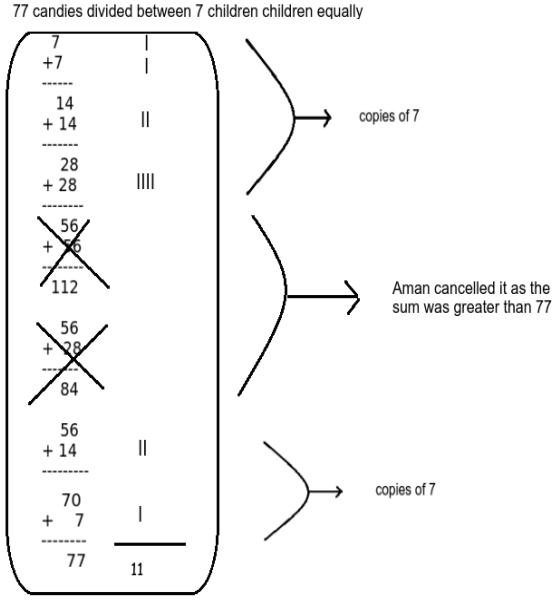
When a group of Class 5 students were given this problem the teacher wasn't sure how the students were going to solve. "*2 pencils and 1 eraser cost Rs. 13 and 1 pencil and 2 erasers cost Rs. 11. What is the cost of 1 pencil? 1 eraser?*"

After a couple of minutes, some children said that the cost of the pencil was Rs. 5 and the eraser was Rs 3. The teacher noticed that whatever calculations the children had done were oral. When asked how they got the answer, one girl said, "*2 pencils and 1 eraser and 1 pencil and 2 erasers give us the cost of 3 pencils and 3 erasers, that is Rs.24. Then we get the cost of 1 pencil and 1 eraser which is Rs. 8 and from that we get the cost of 1 pencil and 1 eraser.*" I was very surprised to hear this. The children were never taught this strategy. They had figured out strategies to '*find the unknown*' themselves.

Though I said that the children were not taught this strategy, I should mention that in this classroom there was a complete freedom for the children to use their own ideas and solve problems. All methods were welcomed and the only condition was that the children were expected to justify their methods to their classmates. Hence the children weren't worried of going wrong.

Whenever I think about children's own methods I think about Aman. One of my colleagues decided to teach mathematics to students who had difficulty in mathematics. The teachers from a school near-by were contacted and some children who the teacher thought were weak in mathematics were asked to come every week. Most of the children lived in a basti close to the school and belonged to the lower socio-economic class of society. Some of them helped their family in earning their livelihood and were very quick and confident of doing oral mathematics.

In these sessions, children were given various problems (mostly contextual) and their methods were discussed in the classroom. During one such session, the children were asked a problem which involved equal division. Aman very quickly solved the problem. The method he used was something like the way given below.



(The numbers Aman actually used might have been different)

When Aman showed us his method, for a few minutes we were unable to understand the method and its working. When we understood the method, all of us appreciated it and told Aman that we liked the method very much. Aman's reaction to this was very surprising, he looked at us very confused and said, "You liked this? My teacher didn't like it at all"

This happened a couple of years ago. Since then I have talked about this method to many teachers and teacher educators. Their reactions have been very varied. But one reaction that remains with me was given by a teacher educator, she refused to believe that this method was 'mathematically' correct as, '*multiplication is repeated addition and division was repeated subtraction*'. Her argument was simple, "How is that no mathematics books have realized that division can be done with adding repeatedly when a 10 year old boy could do it?"

It is surprising that while we talk about Gauss's method of adding first

100 numbers is an effective way when he was in school, we refuse to believe that Aman can think about a new algorithm to divide numbers.

Coming back to the question we started with, 'Is *fear of mathematics* a local issue?' Can we study one teacher, one classroom or one school and decide on the factors which cause this fear of mathematics? My answer to this is No.

This fear of mathematics comes from the curriculum, its disconnect from real life due to the nature of mathematics or the way it is organised. Lack of teacher preparation is a major factor why the students feel the disconnect. Our system expects teachers to teach without giving them the help they need to teach. The NCF 2005 position paper on teaching of mathematics underlines a lot of issues raised in this article and the NCERT textbooks, especially the primary textbooks, do work towards addressing some of the issues stated in the position paper.

Various individuals and organisations have been working very hard to develop trajectories, curriculum and teaching education material through practice and research to help students and teachers to understand mathematics or to teach it in a more effective way.

When I reached the end of this article I wondered why I was writing it. Am I asking some new questions, offering some solutions? Maybe not. But I feel that there are some questions which have to be asked again and again so that we don't forget the issues involved.

I would also like to add that this article in no way wants to question the efforts done by the mathematics education community in India but wants to say that we need to work even more towards a mathematics education which is more equitable and inclusive.

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कक्षा सात के बच्चों के साथ बीज गणित पर बातचीत

सार

बीज गणित की कई अमूर्त धारणाएं बच्चों को इसलिए कठिन लगती हैं, क्योंकि उनके विषय में बच्चों को हम न तो प्रश्न करने का अवसर देते है और न ही उनसे बात करते हैं। इस शोध पत्र में उत्तरकाशी के नॉगुर ब्लॉक के एक स्कूल में किए गए कार्य का विश्लेषण है। यह देखा गया है कि संवाद व खोजने के अवसरों से बच्चे अमूर्त धारणाओं से जूझ पाते हैं। सब बच्चे चर्चा में भाग लेते हैं व योगदान देते हैं यदि उन्हें मौका और माहौल मिले।

मुझे गणित की कार्यशाला के पश्चात कुछ असाइनमेंट्स करने थे जिनमे से एक था कक्षा सात के बच्चों के साथ बीजगणित पर बातचीत। इसके लिए विद्यालय का चुनाव करना था तो जो सबसे पहला और एकमात्र नाम मेरे ज़ेहन में आया वह था उच्च प्राथमिक विद्यालय गोना का जो, कि उत्तरकाशी के नौगाँव ब्लाक के गडोली संकुल में स्थित है। मैंने कक्षा सात में विद्यार्थियों की संख्या पूछी तो पंद्रह बताई गई जो कि मेरे कार्य के लिए पर्याप्त थी।

तैयारी के तौर पर एक कार्ययोजना बनायी जिसमे बच्चों के तीन समूह बनाकर उन्हें कार्य दिया जाना तथा उसके बाद सभी के साथ बातचीत करना तय किया। इस कार्य हेतु तीन वर्कशीट्स बनायी और कुछ सवालों को बुना जिस पर बच्चों के साथ बातचीत शुरू की जा सके। तीनों वर्कशीट्स को बनाने के आधार निम्न थे:

अ) पहले समूह को 'सूरज' नाम दिया तथा उनके लिए जो वर्कशीट तैयार की उसका उद्देश्य यह जानना था कि बच्चे संक्रियाओं के सरलीकरण को कितना समझते हैं, तथा 'अनंत' जैसे शब्दों को कैसे समझते है। यह इसलिए रखा गया क्योंकि अनंत तक की समझ के लिए हमें कुछ मूलभूत नियम या सूत्रों की आवश्यकता पड़ती है जिसे बीजगणित के द्वारा बेहतर समझा जा सकता है।

आ) दूसरे समूह का नाम 'चाँद' दिया तथा उनकी वर्कशीट का उद्देश्य यह था कि बच्चे संक्रियाओं को दैनिक जीवन की समस्याओं से कैसे जोड़ते हैं तथा वे इन समस्याओं के निदान के क्या तरीके सोचते हैं। साथ ही यह भी जानना था कि संख्या के स्थान पर अक्षरों के प्रयोग से वे कितना परिचित हैं।

इ) तीसरे समूह का नाम 'तारे' दिया तथा उनकी वर्कशीट का उद्देश्य यह जानना था कि बच्चे मापन में इस्तेमाल होने वाले सूत्रों को कितना अच्छे से समझते हैं।

इस पूरी गतिविधि एवं बातचीत के लिए बच्चों के साथ एक घंटे के समय का अनुमान लगाया।

पहला दिन:

विद्यालय में पहुँचने पर देखा बच्चे कुछ-कुछ काम कर रहे थे। कुछ ने विद्यालय की सफाई संभाल रखी थी तो कुछ ने पानी भरने की। कुछ बच्चे रास्ते से आ रहे थे और कुछ और मुझे देखकर शरमाते हुए कानाफूसी कर रहे थे। एक बच्चे ने आकर गुरूजी की ओर देखा और उनका इशारा मिलते ही घंटी बजा डाली। कक्षा की घंटी लगी और सभी बच्चे अपनी-अपनी कक्षाओं में चले गए। मैं नीचे कक्षा में पहुंचा तो देखा दस बच्चे थे, जिनमे से सात लड़कियां और तीन लड़के थे, बाकी के पांच बच्चे आज अनुपस्थित थे।

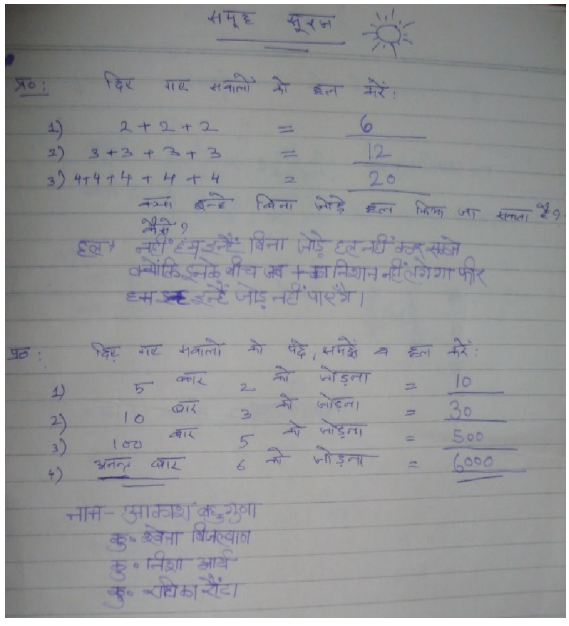
मुझे देखकर कुछ बच्चे पहचान गए और कुछ आँखें चुराकर एक दूसरे को इशारा करने लगे। मैंने बातचीत की शुरुआत यह पूछकर की कि कितने बच्चे मुझे पहचानते हैं, जवाब में तीन बच्चों ने हाथ उठाया। मैंने पूछा मेरा नाम क्या है तो वे बोले, 'जी यश सर जी'। मुझे अच्छा लगा कि इतने लम्बे अंतराल के बाद भी बच्चों ने मुझे और मेरे नाम को याद रखा। इसके बाद मैंने उनका परिचय लिया, जिसपर वो एक-एक कर खड़े होकर, अपने हाथों

को फोल्ड कर एक ही लय में बताते गए, 'जी मेरा नाम काजल राणा है जी', 'जी मेरा नाम मुकेश भट्ट है जी'..... मैंने उनसे पूछा कि किस-किस को गणित अच्छी लगती है? तो सबने अपने हाथ उठाये, रमिता का हाथ नीचे था पर सबने उसको इशारों में समझाया तो उसने भी झेंपते हुए अपना हाथ उठा ही लिया। अब मुझे अपना काम भी करना था, सो मैंने उन्हें बोला, 'चलो एक काम करते हैं तीन समूह बनाते हैं और कुछ-कुछ काम करते है और फिर बातचीत करेंगे, ठीक है?'। बच्चों ने कहा, 'ठीक है जी'। मैंने उनसे बोला तीन समूह बनाने के लिए तो उन्होंने तीन लड़कों का अलग, तीन लड़कियों का अलग और चार लड़कियों का अलग समूह बना लिया। मैंने पूछा ऐसा क्यों बोले, 'जी वो हम साथ ही रहते हैं इसलिए'। मैंने तीनों लड़कों को तीन समूह में बांटा और ऐसे ही लड़कियों को भी बांटकर नए समूह बनाए और पूछा, 'ऐसे में कोई परेशानी तो नहीं होगी?' तो उन्होंने अपने-अपने सिर नहीं में हिलाए। मैंने कहा, 'अब देखो काम क्या करना हैं, मेरे पास तीन कागज़ हैं जिनपर कुछ काम लिखा है जो समूह को मिलकर करना होगा और उसके बाद हम कुछ बातचीत करेंगे। तीनों कागजों के नाम भी हैं; सूरज तारे और चाँद, तो तुममें से कौन क्या लेना चाहेगा?' आकाश बोला, 'जी हमें सूरज' और काजल बोली, 'हमें तारे जी'। तीसरा समूह नहीं बोला क्योंकि उन्हें पता था के उन्हें तीसरा यानि चाँद लेना ही पड़ेगा। अब मैंने तीनों समूहों को अलग अलग वर्कशीट दी, और कहा, 'इसमें दिए गए सवालों को समूह में बात कर हल करें और फिर हम मिलकर बातचीत करेंगे और इसके लिए हमारे पास हैं पन्द्रह मिनट। तीनों समूह सवालों को पढने लगे।

तीनों समूहों ने कुछ इस प्रकार काम किया:

समूह सूरज:

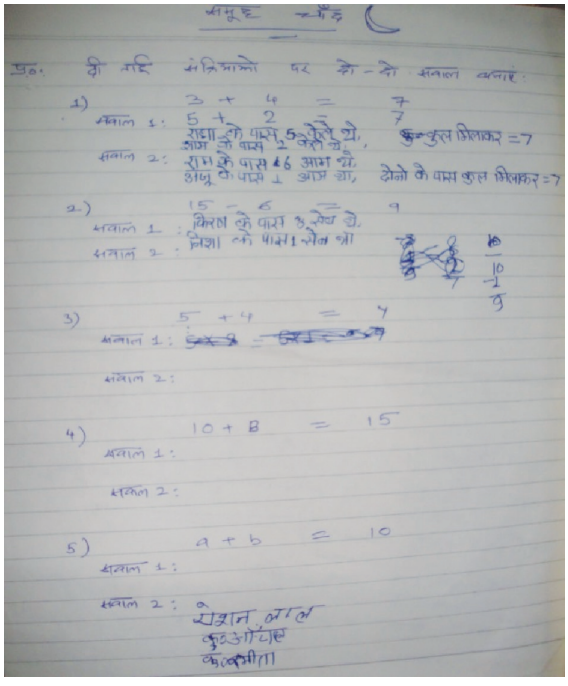
इस समूह की बातचीत को आकाश लीड कर रहा था। उसने सब से सभी सवालों पर राय ली और फिर कागज़ पर भरा। दो सवालों पर उनको थोड़ी दिक्कत हुई; क्या बिना जोड़े सवाल हल नहीं हो सकता और दूसरा अनंत का मतलब क्या होता है? पहले सवाल पर बातचीत में श्वेता ने बोला, 'अरे बिना जोड़े कैसे करेंगे , इसमें तो '+' का निशान भी है'। अब आकाश ने जोड़ा, 'यदि निशान नहीं होगा तो हमें पता कैसे चलेगा कि करना क्या है, इसलिए हमें इस सवाल को जोड़कर ही करना होगा'। दूसरे सवाल पर आकाश ने अपनी टिप्पणी दी, 'अनंत का मतलब होता है सबसे बड़ा, और सबसे बड़ा होता है हज़ार, तो यहाँ पर उत्तर आएगा छह गुना हज़ार यानि छह हज़ार'। इस पर सबने सहमति में अपना सिर हिलाया और आकाश ने उसे कागज़ पर लिख दिया।



समूह चाँद:

इस समूह को लीड करने का काम आँचल ने लिया और उसने सवाल पढना शुरू किया। यहाँ पर संक्रियाओं शब्द पर समस्या आ रही थी और ये तय करना मुश्किल हो रहा था कि करना क्या है। रमिता जो की बाकी के दो समूहों को देखे जा रही थी बार बार बोल रही थी, 'जल्दी लिख ना, वरना हम पीछे रह जायेंगे'। पहले सवाल पर आँचल बोली, 'तीन और चार सात तो पांच और दो भी तो सात होते हैं'। बाकि दोनों बोले लिख-लिख। सो आँचल ने लिख दिया। अब आँचल, रोशन और रमिता फिर फुसफुसाने लगे। मैंने उनसे आखिरकार पूछ ही लिया, 'क्या हुआ सवाल समझ नहीं आ रहा है क्या?'। इस पर वो बोले 'जी सर जी'। मैंने उन्हें बताया, 'तुम्हे इन सवालों पर सवाल बनाने हैं। जैसे 3+4=7, का सवाल हो सकता

है कि यश के पास यदि तीन टाफ़ियाँ हैं और रमिता ने उसे चार और दे दी तो उसके पास कुल सात टाफ़ियाँ हो जाऐंगी। कुछ ऐसे ही सवाल बनाने हैं, समझे'। वो बोले, 'जी सर जी'। उसके बाद वो कुछ छुपा छुपा कर लिखने लगे।



समूह तारे:

काजल ने इस समूह को सँभालने का जिम्मा लिया। उसने सवाल पढ़ा और मुझसे बोली, 'सर जी ये कहाँ की लम्बाई नापनी है?' तो मैंने कहा, 'एक काम करो बरामदे को नाप लो'। इतना सुनकर उसने सबको बोला, 'चलो बाहर चलें'। बाहर जाकर उन्होंने बरामदे को कदमों से नापना शुरु किया, इतने में मुकेश बोला, ' हमारे पास तो फीता ही नहीं है तो हम कैसे नापेंगे? चलो एक काम करते हैं, हम इस लकड़ी को तोड़ कर इस से नापेंगे'। बबीता बोली, 'हाँ ये ठीक है पैरों से बराबर नाप नहीं आता है'।इसके बाद वे सभी कक्ष में आये और दिए गए प्रश्नों पर जवाब लिखने लगे। बरामदे की लम्बाई और चौडाई लिखने के बाद तीसरे प्रश्न पर वो अटक गए। प्रश्न था, 'लम्बाई और चौड़ाई की मदद से आप फर्श से सम्बंधित कौन-कौन सी जानकारी ज्ञात कर सकते हैं? तथा कैसे?'। इस पर सभी ने बात करी और मुकेश बोला, 'फर्श की लम्बाई गुणा चौड़ाई लिख दे'। काजल ने लिख दिया फिर उसने अगले प्रश्न को पढ़ा जो उनको समझ नहीं आया। और समय समाप्त होने का आभास उन्हें मैं करा चुका था। सो उन्होंने दायें और बाएं देखा और बबिता बोली, 'हमने सबके जितना लिख दिया है, बहुत है बस करो'।

मैंने ये तय किया था के इस गतिविधि के बाद मै बड़े समूह में कुछ प्रश्न करूंगा:

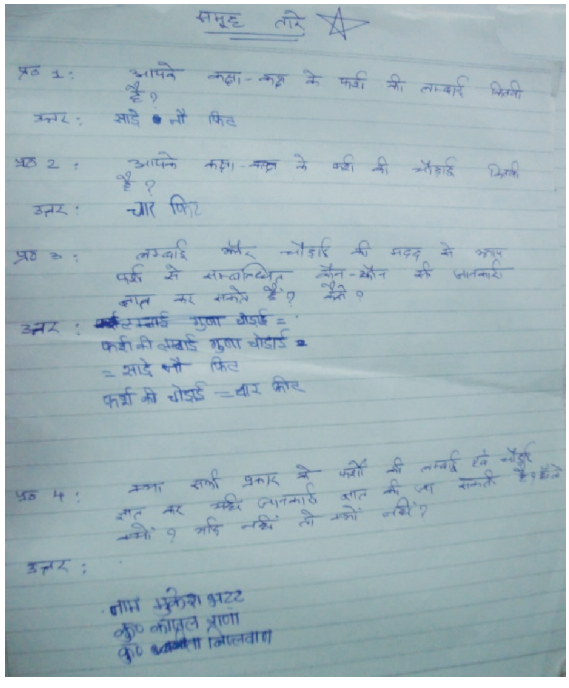
अ) हमने संख्या के बदले अक्षर क्यों लिखे?

ब) क्या 2xM और 2M में कोई अंतर होता है?

इ) 2A+3B का मतलब क्या होता है?

ई) बीजगणित क्या होती है?

परन्तु स्थिति को देखते हुए मैंने बातचीत कर इस गतिविधि के प्रश्नों पर चर्चा करना तय किया।



समय समाप्त हुआ और मैंने सभी को एक साथ अर्धवृताकार बैठने को कहा। अब मैंने पूछा, 'कैसा लगा इस काम का करने में?' तो सभी मिलकर बोले, 'जी अच्छा जी'। मैंने बात आगे बढ़ाई और हर समूह को एक-एक कर अपने प्रश्न और उसके उत्तर पढने को कहा। सबसे पहले समूह सूरज ने पढना शुरू किया।उनके समाप्त करने पर मैंने सभी से पूछा, 'अच्छा ये बताओ 'अनंत' का

मतलब क्या होता है?'। आकाश बोला, 'जी सबसे बड़ा'। मैंने पूछा, 'तो सबसे बड़ी संख्या कौन सी हुई?' आकाश बोला, 'जी हज़ार जी', इसपर काजल बोली, 'नहीं जी हज़ार से बड़ा तो लाख होता है'। इस पर सबने जोड़ना शुरू किया, जी लाख से बड़ा करोड़ और करोड़ से बड़ा अरब होता है'।एक और ने जोड़ा, 'सबसे बड़ा तो शंख और महाशंख होता है'। मैंने पूछा, 'कैसे पता?' तो बोले, 'जी सर जी ने बताया है जी'। अब मैंने पूछा, 'महाशंख में कितने शून्य होते हैं?' तो सभी गिनती करने लगे और फिर आकाश बोला, 'जी पंद्रह'। मैंने कहा, 'क्या कोई संख्या 1000000000000000 से बड़ी हो सकती है'। 'जी नहीं सर जी', मैंने बोला, 'क्या 1000000000000000 में 1 को जोड़ सकते हैं?' तो कुछ मिलकर बोले, 'जी नहीं जी'। मैंने कहा, 'अच्छा तो दस में एक जोड़कर क्या बनता है?' वो बोले जी ग्यारह। मैंने फिर पूछा, 'सौ में एक जोड़कर?' वो बोले जी एक सौ एक।मैंने कहा कि ऐसे ही लाख में एक जोड़कर क्या बनेगा? वो बोले जी एक लाख एक। इस पर मैंने पूछा, 'तो अब बताओ महाशंख में एक जोड़ेंगे तो क्या बनेगा?' वो बोले जी महाशंख एक। मैंने पूछा, 'तो महाशंख बड़ा हुआ या महाशंख एक?' वो बोले जी महाशंख एक। 'तो क्या महाशंख सबसे बड़ी संख्या हुई?', जी नहीं जी बच्चों का जवाब था।तो मैंने बोला, 'तो इसका मतलब कोई भी संख्या सबसे बड़ी नहीं हो सकती क्योंकि उसमे एक के जुड़ने की संभावना रहती है'।'यानी के अनंत संख्या का मतलब वो संख्या हुई जिसके आगे के शून्यों की कोई सीमा नहीं होती है'। सबने अपने सिर सहमति में हिलाए। मेरा एक काम पूरा हुआ, मैंने अब उनसे कहा, 'अगर 6 को अनंत बार जोड़ेंगे तो क्या मिलेगा?' वो बोले जी कुछ नहीं। मैंने कहा क्या मतलब? तो बोले 6 को अनंत बार जोड़ने से कुछ नहीं मिलेगा क्योंकि अनंत को लिख तो सकते नहीं हैं'। इस पर मैंने कहा, 'मान लो B एक अनंत संख्या है तो 6 गुना B क्या होगा?' तो वे बोले जी 6xB। 'ठीक है, अब आगे बढ़ते हैं'। समूह तारे से काजल ने ही सवालों को पढ़ा और उनके उत्तर भी पढ़े। इनसे मैंने तीसरे प्रश्न के बारे में पूछा, 'लम्बाई और चौड़ाई की मदद से आप कौन-कौन सी जानकारी ज्ञात कर सकते हो? इसपर काजल ने कहा, 'जी लम्बाई गुना चौड़ाई'। 'तो लम्बाई गुना चौड़ाई से क्या निकलता है?' मैंने पूछा। 'जी क्षेत्रफल' इस बार आँचल बोली। 'तो और क्या क्या निकल सकता है?' सभी शांत होकर बैठ गए। बड़ा गहरा सन्नाटा था... मैंने हंसते हुए बोला, 'इतना सन्नाटा क्यों है भाई? अरे मै तो ये जानना चाह रहा था कि क्या तुमने दो गुना लम्बाई धन चौड़ाई कही पढ़ा है?'। 'जी सर जी, परिमाप में' आकाश बोला। 'तो इसका मतलब हुआ कि हम लम्बाई और चौड़ाई की मदद से क्षेत्रफल और परिमाप निकाल सकते हैं?', मैंने कहा। 'जी' सब मिलकर बोले।'चलो आगे बढ़ते हैं तीसरे समूह चाँद ने क्या किया है उनसे सुनते हैं' मैंने कहा।

चाँद समूह से आँचल ने ही सवालों को पढ़ा और जवाब भी। अब मैंने बड़े समूह से पूछा, 'अच्छा क्या तुम लोग भी 5+7 पर सवाल बना सकते हो?'।'जी सर जी' कुछ आवाजें आई।'अच्छा तो क्या तुम लोग ये बता सकते हो कि जैसे 3+4=7 का मतलब था कि किसी के पास तीन चीज़ें थी और किसी और के पास चार तो दोनों को मिलाकर कुल सात चीज़ें हो गयीं, इसी तरह 10+B=15 का मतलब क्या होगा?'। सभी सोच में पड़ गए। मैंने कहा, 'चलो ये मैं बताता हूँ, आँचल के पास दस सेब हैं और मेरे पास B सेब, और हम दोनों के पास कुल मिलाकर पंद्रह सेब हैं। 'अब तुम बताओ इसी तरह a+b=10 का मतलब क्या हो सकता है?'। इसपर आकाश थोडा सा सहमकर बोला, 'जी मेरे पास a सेब हैं और आपके पास b और हम दोनों के पास कुल मिलाकर दस सेब हैं'।'बहुत अच्छे आज के लिए इतना बहुत है, कल फिर मिलेंगे, ठीक है?' 'जी सर जी, कल आप फिर से आना'।

दूसरा दिन: आज बच्चों की संख्या कल से कम थी और खासकर कक्षा सात के जाने पहचाने कुछ ही चेहरे दिख रहे थे। मै कक्षा कक्ष में पहुंचा, तो देखा आज, कल वाले सभी बालक अनुपस्थित थे; रोशन, आकाश और मुकेश। पूछा तो बालिकाओं ने पता नहीं के भाव से जवाब दिया। खैर मैंने सात बालिकाओं के साथ ही बातचीत शुरू करना तय किया।

मैंने उनसे पूछा, 'अगर मेरी उम्र राधिका की उम्र से तीस वर्ष अधिक है और यदि राधिका की उम्र बारह वर्ष है, तो बताओ मेरी उम्र कितनी है?'। थोड़ी देर एक दूसरे से बात कर आँचल बोली, 'जी बयालीस वर्ष'। मैंने कहा कैसे तो बोली, 'जी वो बारह में तीस जोड़ कर बयालीस ही आता है'।मैंने कहा, 'अगर इसे कॉपी पर हल करना हो तो कैसे करेंगे?'। मैंने कॉपी उनके आगे बढ़ा दी तो उनमे से एक ने ऐसे लिखा:

12+30=42

मैंने पूछा, 'क्या इस तरह के इबारती सवालों को किसी और तरीके से लिख सकते हैं?'। शायद उनको मेरा सवाल समझ नहीं आया। मैंने उनसे कॉपी ली और उसपर कुछ ऐसे बोलते हुए लिखा:

मेरी उम्र = राधिका की उम्र + 30
= 12 + 30
= 42 वर्ष

तो इसपर सभी बोले, 'जी ठीक है'। मैंने फिर पूछा, 'क्या इसे किसी और तरीके से कर सकते हैं?' इस बार फिर सन्नाटा सा छा गया। मैंने फिर कॉपी में बोलते हुए लिखना शुरू किया, सभी के सिर जुड़कर कॉपी के ऊपर आ गए। मैंने लिखा:

माना मेरी उम्र = a वर्ष
राधिका की उम्र = b वर्ष
तो,
a = b + 30

'क्या ये सही लिखा है?' मैने पूछा। वो बोले, 'जी सर जी'। मैंने आगे लिखा:

चूँकि b = 12 वर्ष
तो, a = 12 + 30
a = 42
यानि, मेरी उम्र = a वर्ष = 42 वर्ष

मैंने कहा, 'समझ में आया कुछ?' वो बोलीं, जी सर जी'। इसके बाद मैंने एक सवाल और हल करके दिखाया, सवाल था:

आँचल की ऊंचाई काजल की ऊंचाई से दो सेंटीमीटर अधिक है। यदि आँचल की ऊंचाई 122 सेंटीमीटर है तो बताओ काजल की ऊंचाई कितनी होगी?

अब मैंने एक सवाल बोला और उनसे हल करने को कहा। सवाल था:

यश के पास रमिता से पचास रुपये अधिक हैं और श्वेता के पास रमिता से बीस रुपये कम हैं। यदि रमिता के पास तीस रुपये है तो यश और श्वेता के पास कितने रुपये हैं?

इसपर उन्होने आपस में बातचीत कर कॉपी में ये लिखा:

यश के पास = 50 रूपये
श्वेता के पास = 10 रुपये
रमिता के पास = 30 रुपये

मैंने उनसे पूछा, 'ये उत्तर कैसे आया?' तो आँचल बोली, ' सर जी रमिता के पास तीस रुपये तो श्वेता के पास बीस रुपये कम यानि दस रुपये होंगे'। मैंने कहा, 'और यश के पास पचास कैसे?' तो वो बोली, 'अम्म्म पता नहीं ज्यादा होंगे'। मैंने एक बार फिर उन्हें सवाल हल करके दिखाया। मुझे लगा उन्हें समीकरण हल करने में दिक्कत हो रही है सो मैंने उन्हें सरल समीकरण हल करवाए। जैसे:

a – 3 = 6, –2 + b = 8, 5 + p – 3 = 6 आदि।

अब मैंने आगे बढ़ने के लिए नया सवाल पूछा, 'अच्छा अब ये बताओ कि एक सौ धन दो सौ बराबर कितना होता है?' उन्होंने मिलकर कहा, 'जी तीन सौ'

मैंने पूछा, 'तो दो सौ धन एक सौ बराबर कितना होगा?' वो बोले, 'जी तीन सौ ही होगा'

ऐसे ही मैंने एक और सवाल पूछा, 'तीन धन दो बराबर कितना?' तो वो बोले, 'जी पांच'

'और दो धन तीन?' मैंने पूछा

'जी पांच ही होगा' वो बोले

मैंने कहा, 'तो इसका मतलब मै तीन धन दो करूँ या दो धन तीन दोनों के उत्तर बराबर ही आयेंगे यानी पांच। क्या मै ठीक बोल रहा हूँ?'

आँचल बोली, 'जी सर जी, उल्टा पुल्टा कर जोड़ तो एक ही आएगा।

'अच्छा, तो अगर मै पूछूं तीन ऋण दो बराबर कितना?' मैंने कहा
'जी सर जी एक' मिलकर वो बोले
मैंने पूछा, 'तो दो ऋण तीन कितना होगा?'
रमिता बोली, ' जी एक' और आँचल, निशा और राधिका बोलीं, 'जी ऋण एक होगा जी।
और फिर वो मिलाकर रमिता को समझाने लगी और फिर रमिता बोली, 'जी सर जी ऋण एक होगा।

'पक्का पक्का बताओ ऋण एक होगा या एक होगा?' मै बोला
'जी ऋण एक ही होगा' वो सब मिलकर बोली

मैंने कहा, 'तो एक और ऋण एक दोनों अलग-अलग होते हैं या एक ही बात है'

'जी नहीं सर जी दोनों अलग-अलग होते हैं। वो बोले 'यानी के हम ये कह रहे हैं कि जोड़ने में उल्टा पुल्टा चलता है पर घटाने में नहीं। मैंने कहा
वो बोले, ' जी सर जी'
अब मैंने कॉपी पर लिखा

$a + b = b + a$

$a - b = b - a$

मैंने ऋणात्मक संख्याओं के बारे में चर्चा जानबूझकर आगे नहीं बढ़ाई क्योंकि अभी के लिए इतना ही काफी था कि वो ऋण एक और एक को अलग-अलग समझें

'क्या मै इसको ऐसे लिख सकता हूँ?' मैंने पूछा
'जी सर' वो बोलीं।
'तो फिर ये तो एक नियम बन गया जो कि सब पर लागू होगा।' मैंने कहा
'जी सर जी' वो बोले।
'नियम का अर्थ हमने देखा कि एक ऐसा कथन जो सभी परिस्थियों में लागू होता हो। यानी हमने जो नियम निकाला वो योज्य प्रतिलोम का नियम है जो कि ऋण प्रतिलोम में लागू नहीं होता है। इसी प्रकार हम जब किसी नियम का प्रयोग किसी भी प्रकार के मान को ज्ञात करने के लिए करते हैं तो उसे सूत्र कहते हैं।उदाहरण के लिए परिमाप का सूत्र या क्षेत्रफल का सूत्र या आयतन का सूत्र आदि।'

आज के अपने काम को देखें तो हमने तीन बातों पर चर्चा करी:

- अज्ञात राशि को ज्ञात करना जो कि हमने शुरू के काम में किया जहाँ यश के पास कुछ पैसे और रमिता के पास कुछ वाले सवाल थे।
- नियम बनाना जिसमें हमने $a + b = b + a$ के नियम को समझा
- और ये भी जाना कि नियम और सूत्र व्यापक रूप से प्रदर्शित करने में काम आते हैं
- आज और कल में हमने जो काम किया उसमें निम्न बातें थी:
- संख्याओं के स्थान पर अक्षरों का प्रयोग
- अक्षर संख्याओं का मान किसी परिस्थिति में तय या एक ही मान होना परन्तु किसी अन्य परिस्थिति में एक से अधिक मान की भी संभावना
- अक्षर का व्यापकीकरण करने के लिए उपयोग करना
- व्यापकीकरण नियम एवम सूत्रों द्वारा करना

ये सभी बिंदुओं का अध्यन 'बीजगणित' के अंतर्गत आता है।

'सर जी बीजगणित तो हमने पढ़ी थी, हमारे सर जी ने भी पढ़ाई थी।' आँचल बोली
'तो ठीक हैं हमने उसे और बेहतर से समझने की कोशिश की। चलो अब कुछ और करते हैं पर पढ़ाई नहीं। क्या करें?' मैंने कहा
'सर जी पिक्चर दिखा दो, आप से पहले जो आये थे उन्होंने भी दिखाई थी। वो बोलीं

मैंने बोला, 'ठीक है, आखिरी घंटे में हम सभी मिलकर एक छोटे से बच्चे की पिक्चर देखेंगे। अंतिम घंटे में हमने मिलकर 'Do Flowers Fly' मूवी देखी, जिसको सभी बच्चों के साथ-साथ अध्यापकों नें भी पसंद किया। बच्चों ने तो अपने अनुभव भी लिखे और साझा करे।

नोट: इस कार्य को करने के पश्चात दस्तावेजीकरण करने में लगभग दो महीनें का समय लगा है। इसलिए कुछ वार्तालाप में अनुमान के आधार पर बातें रखी गयी है परन्तु प्रक्रिया के दस्तावेजीकरण में किसी भी प्रकार से नयी बातों को नहीं जोड़ा गया है व उसके प्रमाण लिखने वाले के पास मौजूद हैं।

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Planning In-service Training Programme: Report of a Need Assessment Survey

Abstract

The present paper explains about 'why' and 'how' of assessing training needs of teachers for finalising the structure of professional development programmes with the help of empirical data. A Need Assessment Questionnaire (NAQ) has been administered to a sample of primary school teachers to collect information regarding the requirements of in-service training in the area of teaching primary mathematics. The analysis of the data shows that teachers require more orientations on student-centred, activity-based learning methods which are appropriate to the primary school level.

Introduction

The National Curriculum Framework (NCF) 2005 highlights the importance of Mathematics and stated that the main goal of Mathematics education is mathematisation of the child's thought processes. Since mathematical understanding influences decision making in all areas of life, it is considered as the most important of all curriculum subjects. All the major commissions and committee reports on education since independence rightly emphasised the importance of mathematical knowledge and its utilitarian values. In spite of all these reports and recommendations still in India, many students still struggle with Mathematics and show disinterest in learning Mathematics. The National Achievement Surveys of NCERT being conducted time to time clearly bring out this declining trend in Mathematics over the years. The same is the case with board exam results in Mathematics of different states and central boards in India.

A number of factors may influence the learning of Mathematics but teachers play an important role in the performance in Mathematics. The knowledge in Mathematics alone will not help a person to teach Mathematics. He/she needs to have sound knowledge in the area of teaching of Mathematics. The knowledge in Mathematics and how to teach Mathematics together is commonly known as Pedagogical Content Knowledge (PCK).

Primary Mathematics, being the base for later stage, is very crucial in mathematisation of child's thought process. If we are not able to provide opportunities to our young children to experiment with mathematical concepts, formulae, principles etc, we may not be in a position to realise the goal of mathematisation. Let me share my experience with students when I was teaching in a school as a part of a three month field visit programme.

A girl who was considered as excellent in all subjects including Mathematics performed a mathematical operation during one of the problem solving session in the following manner:

"113 - 64/113 - 49 = -64/-49 = 64/49 (By cancelling 113 from numerator and denominator)

Another student when asked to measure the three interior angles of a triangle using protractor measured the angles like this.

"∟A= 68°, ∟B= 139° and ∟C= 111°" What is wrong with these children? Can we say, this is due to the problem of child alone? As teachers are we responsible for these types of errors and misconceptions? Are we providing need based professional development programmes to the teachers? Having been confronted with these types of situations, author thought of developing a comprehensive plan for organising professional development programmes for various stake holders like teachers and teacher educators.

Professional development of teachers is central to improving the quality of education in schools. The way we organise these programmes also are equally important for realising the expected goals. It was felt that simply organising an in-service programme will not serve the purpose. In order to improve the basic mathematical abilities of our primary children, the first stage to be done is to understand the basic components to be included in the programme material. For this purpose, author has developed a questionnaire for assessing the training needs of teachers in various areas.

The Need Assessment Questionnaire (NAQ) has been administered to a sample of 100 primary school teachers. The responses were received from only 84 teachers and the same have been used for analysis and interpretation. The basic purpose of the NAQ was to collect information regarding the requirements of in-service training in the area of teaching primary Mathematics.

The major aspects included in the questionnaire are:

- Pedagogical practices being followed in the classroom.
- The current practices of student assessment.
- Active participation of student.
- The broad areas like content, pedagogy, child psychology, assessment, etc. to be covered in professional development programme.
- The content/topic/theme from primary Mathematics in which further orientation is required.
- Duration and modality of in-service training programme.

The purpose of the needs analysis was to identify the needs and requirements of primary school teachers in the area of content, pedagogical approaches, assessment procedures, etc. for developing the training package for using in the in-service training programme. The data collected through NAQ was analysed using percentage and is presented in the following subsections.

Pedagogical practices followed in the classroom

Pedagogical approaches practiced by the teachers has to play crucial role in making the subject interesting or boring. To the question why most of the students show fear towards Mathematics, can be best answered with the help of pedagogical practices of the teacher. It is true that a teacher can make a big difference. In order to understand the practices followed by the teachers in their classroom transaction, the following question with 11 alternative strategies that could make the classroom process vibrant and constructive were posed to the teachers.

About how often do you do each of the following in your Mathematics instruction?

S. No	Aspects	Rarely	Sometimes	Most times	Always
1.	Introduce content through formal presentations	14(16.7)	16(19.1)	51(60.7)	3(3.6)
2.	Facilitate students individually and/ or group during various classroom activities	11(13.1)	21(25)	46(54.8)	6(7.1)
3.	Pose open ended questions	16(19.1)	32(38.1)	33(39.3)	3(3.6)
4.	Engage the whole class in discussions	4(4.8)	19(22.6)	49(58.3)	12(14.3)
5.	Ask students to explain concepts to one another	19(22.6)	28(33.3)	31(36.9)	6(7.1)
6.	Ask students to consider alternative methods for solutions	25(29.8)	27(32.1)	28(33.3)	4(4.8)
7.	Allow students to work at their own pace	15(17.9)	38(45.2)	23(27.4)	8(9.5)
8.	Help students see connections between Mathematics and other disciplines	19(22.6)	42(50)	21(25)	2(2.4)
9.	Assign Mathematics homework which helps to develop creativity	14(16.7)	34(40.5)	28(33.3)	8(9.5)
10.	Give tests requiring open ended responses (e.g., descriptions, explanations)	9(10.7)	17(20.2)	42(50)	16(19.1)
11.	Link mathematical concepts with children's lives	17(20.2)	39(46.4)	22(26.2)	6(7.1)

The responses given by the participants portraits the current situation of our Mathematics classroom. As mentioned earlier also, motivating students to ask more and more thought provoking question, is an important pedagogical strategy needs to be practiced by the teachers. The response shows that more than 57% of teachers use this either rarely or sometimes only.

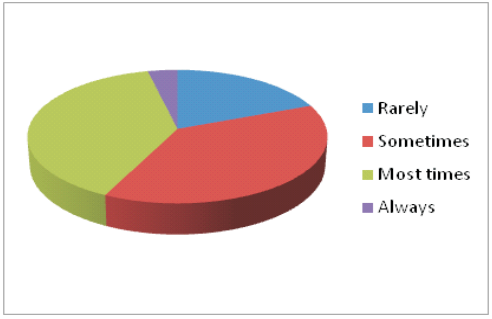

Figure 1: How often teachers pose open ended questions

Creativity is an outcome of divergent thinking. If we are not giving opportunity to the child to provide alternative pathways, the divergent thinking will not happen. Same problem can be solved in different ways. How far the teachers are efficient to provide situations to the child to think about alternative perspectives is an important component for Mathematics learning? The response to this question shows that around 62% of the teachers responded were only did this either rarely or sometimes in their classroom. That is only 38% of teachers are practicing this approach in their classroom seriously.

Students like Mathematics if they get ample opportunity to connect Mathematics with their real life situations. The response of the teachers towards this question also shows that most of the teachers (66%) are not linking mathematical concepts with child's life or other discipline (57%).

What more is required is that the teachers need to get more and more opportunities for improving their abilities to use collaborative and constructivist approaches in the classroom.

The current practices of student assessment

Continuous and Comprehensive Evaluation (CCE) of students leaning is still a big challenge to many of the teachers. In fact, many central and state level institutions had organised orientation programmes to teachers at various levels. Still the implementation of CCE in majority of our classrooms is considered synonyms to the process of completing/filling various forms and schedules. This subsection in this survey focuses on the implementation of assessment strategies in the classroom and the areas of concerns to be addressed in the training programme.

How often do you assess student progress in Mathematics in each of the following ways?

S.No	Statement	Rarely	Sometimes	Most of the times	Always
1.	Previous knowledge checking to determine what students already know	0(0)	5(6)	38(45.2)	41(48.8)
2.	Observe students and ask questions as they work individually in each period	37(44.1)	44(52.4)	3(3.6)	0(0)

3.	Observe students and ask questions as they work in small groups in each period	23(27.4)	35(41.7)	17(20.2)	9(10.7)
4.	Motivate students to ask questions	49(58.3)	23(27.4)	8(9.5)	4(4.8)
5.	Use integrated assessment strategies in class activities	6(7.1)	15(17.9)	37(44.1)	16(19.1)
6.	Review student homework every day	3(3.6)	6(7.1)	47(56)	28(33.3)
7.	Review student portfolios	28(33.3)	29(34.5)	14(16.7)	6(7.1)
8.	Give predominately written tests (e.g., multiple choice, true/ false, fill in the blanks)	4(4.8)	8(9.5)	33(39.3)	29(34.5)

Responses of the teachers to these questions reflect their actual practice in the classroom. Mathematics learning requires constant support from the teacher. Individual attention of the teachers is very essential for the weak students. In this circumstance, teacher observation during individual problem solving situation as well as performing group activities play an important role in building confidence among children. In fact, observation can be considered as an important tool for formative assessment. But the data from the above table shows that most of the teachers use these strategies in their classroom rarely or sometime together (96% for individual observation and 69% group observation respectively). This in fact throws light on the need of more practical oriented capacity building programmes to the teachers for implementing observation as a tool for assessing student performance.

Constructivist classroom warrant more questions from the students. It is the responsibility of the teachers to motivate the students to come up with more and more questions. The creativity and critical thinking ability of the child will improve, if we can offer opportunity to question to our students. The data given shows how far the teachers responded in this questionnaire utilised this strategy in their classroom. 85% of the teachers either rarely or sometimes used it in the classroom. This is a pertinent area of concern.

Student portfolio consists of the collection of various classroom related activities and works. Mere collection will not serve the purpose of assessment. How teachers are assessing this and providing appropriate feedback to the students are very crucial. More than two-third of the teachers (67%) responded that they reviewed the portfolios of the students either rarely or sometimes.

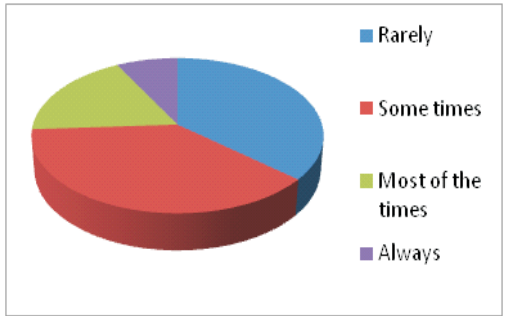

Figure 2: How often teachers review student portfolios

CCE advocates different strategies for student assessment apart from traditional written tests. One of the objectives of CCE is to reduce the examination phobia. Instead, in the

name of CCE if we organise more and more written tests, it will defeat the very purpose of CCE. The response given by the teachers shows that around 74 % of teachers uses written tests either most of the times or always during the assessment. This indicates that they seldom practice the other assessment strategies.

Active participation of student

In a constructivist classroom student should be more active and vibrant, they need to get chance to discuss, perform and ask question. This section discusses how far the classroom facilitates in providing opportunities to our children.

encouraging. Most of the teachers are practicing this either most of the times or always except in one aspect. Independent thinking is an important process through which one can address any issue without the support of anybody. How far the teachers are able to provide learning situations to the students to think independently is paramount important. The response provided by the teachers shows that working individually or in small group without the assistance from the teachers were not practiced most of the times in the classroom.

In Mathematics class, how often do your students do the following?

S. No	Statement	Never or almost never	Sometimes	Most of the times	Always
1.	Work individually without assistance from the teacher	16(19.1)	29(34.5)	32(38.1)	7(8.3)
2.	Work individually with assistance from the teacher	10(11.9)	27(32.1)	37(44.1)	10(11.9)
3.	Work together as a class with the teacher teaching whole class	4(4.8)	21(25)	41(48.8)	18(21.4)
4.	Work together as a class with students responding to one another	13(15.5)	27(32.1)	32(38.1)	12(14.3)
5.	Work in pairs or small groups without assistance from the teacher	26(30.9)	32(38.1)	26(30.9)	0(0)
6.	Work in pairs or small groups with assistance from the teacher	15(17.9)	21(25)	42(50)	6(7.1)

Teachers need to ensure active participation of students in the classroom process. The success of the teacher by and large depends how effectively teacher will transact curriculum with the active involvement of students. The responses given by the teachers in this area are quite

The broad areas like content, pedagogy, child psychology, assessment, etc. to be covered in professional development programme

When we talk about pedagogical content knowledge, one should consider its

various components. First one of course will be the content knowledge. Teachers naturally may not have much problem in this area since most of them are graduates or post graduates and content of primary Mathematics will not be difficult obstacles for teachers. For them the barriers may be in the other areas like pedagogical knowledge. Only awareness about some methods of teaching alone will not serve the purpose. One should know which strategies are better for this class or to the other class, etc. The strategy you are using for one class may not be suitable for the other class. Here one should have enough knowledge about child behaviour. This subsection discusses about the training needs of teachers in various components of pedagogical content knowledge.

Table gives the data collected about the professional development requirements of teachers in are as that are essential for a constructivist teacher. To the statement 'Learning how to use inquiry/ investigation-oriented teaching strategies', more than 50 % of the teachers responded that they require more exposure in this aspect.

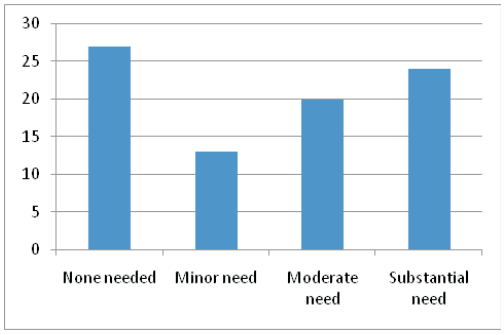

Figure3: Need for Professional development programme in learning how to use inquiry/ investigation-oriented teaching strategies

How would you rate your level of need for professional development in each of the following?

S. No	Areas	None needed	Minor need	Moderate need	Substantial need
1.	Mathematics Content Knowledge	50(59.5)	21(25)	9(10.7)	4(4.8)
2.	The psychology behind developing the students' understanding	42(50)	16(19.1)	18(21.4)	8(9.5)
3.	Learning how to use inquiry/ investigation-oriented teaching strategies	27(32.1)	13(15.5)	20(23.8)	24(28.6)
4.	Learning how to use technology in Mathematics instructions	18(21.4)	25(29.8)	26(31)	15(17.9)
5.	Learning how to assess student learning in Mathematics	48(57.1)	14(16.7)	18(21.4)	4(4.8)
6.	Learning how to teach Mathematics in a class that includes students with special needs	19(22.6)	11(13.1)	23(27.4)	31(36.9)
7	National Curriculum Framework-2005	37(44.1)	13(15.5)	21(25)	13(15.5)

Other areas, in which more than 50% teachers suggested the need of a training is 'learning how to use technology in Mathematics instructions' and 'learning how to teach Mathematics in a class that includes students with special needs'. Inclusion being the policy of the government to implement in a better way, we need to prepare the teachers to face the challenges of providing care and support to all students effectively.

The content/topic/theme from primary Mathematics in which further orientation is required

It may not be possible to discuss all content topics from primary Mathematics in an in-service programme. Being graduates and post graduates it may not be required to organise content specific in-service training programme in all topics. But it is still relevant that some of the teachers may have problems while teaching particular content /theme. This section examines the need of the teachers regarding the in-service programmes to focus on certain content areas/ themes from primary Mathematics. The teachers were asked to suggest the topics from primary syllabus in which they feel some special improvement programmes are required. The table given below gives the data in terms of the number of teachers they require for further improvement in those areas.

Which of the following topics in Mathematics at primary level do you feel need further improvement is required?

S.No	Topics	No of Teachers responded the necessity of further improvement	Percentage
1.	Whole numbers- Counting, Notation, Place Value, Ordering, etc	12	14.3
2.	Concept of Zero	43	51.2
3.	Basic operations on Whole numbers (+,-,,/)	8	9.5
4.	Multiples of a number	8	9.5
5.	Factors of a Number	17	20.2
6.	Fractions and basic operations	32	38.1
7.	Ordering of fractions	37	44.1
8.	Money	30	35.7
9.	Data handling- Classification	37	44.1
10.	Data presentation	36	42.9
11	Data interpretation	39	46.4
12.	Understanding Different Patterns	14	16.7
13.	Measurement of Length, Mass and Volume	36	42.9
14.	Measurement of Time	36	42.9
15.	2 D shapes	17	20.2

16	Angle and its Measurement	35	41.7
17.	Perimeter of simple shapes	17	20.2
18.	Area of simple shapes	20	23.8

The responses given by the teachers presented in the above table give us an idea about the primary school teachers' needs in content specific training. More than half of the teachers express their desire to have more content improvement programme in the topic 'concept of Zero'. The topics in which more than 40 percent teachers require training programmes are Ordering of factors, Data handling, Data Presentation, Data Interpretation, Measurement of Length, Mass and Volume, and Measurement of Time. As a first step we thought of taking these topics for developing training package. The training package will be an integration of the content appropriate to pedagogical strategies for its transaction and coherent strategies for student assessment on a continuous basis.

Duration and modality of in-service training programme

There are different modalities for organising in-service pro grammes. This section describes the opinion of the participant towards the modalities to be followed in in-service training and its duration.

How would you like the in-service training to be delivered per year?

S.No	Mode of Training	No of Teachers*
1	Face-to-face long duration professional development programmes (a period of more than ten days)	16(19)
2	Face to face Short duration professional development programmes (up to five days)	47(55.9)
3	Online	21(25)
4	Blended(online cum face to face)	35(41.7)
5	Any other (2-3 days)	12(14.3)
6	Any other (10 days face to face)	15(17.9)

***Many teachers opted more than one response**

To assess the preferences in terms of different modalities of in- service training, the teachers were asked to indicate their likeness against each alternative. Face to face short duration programmes up to five days turned out to be by far the most popular method. They were chosen by 47(55.9%) of the respondents. 35(41.7%) of respondents selected blended learning, that mixes conventional face-to-face methods and online components.

How often would you like to receive professional training?

S.No	Frequency	No of Teachers* (Percentage)
1	Every Quarter	4 (4.8)
2	2 times/year	7(8.3)
3	Once/year	11(13.1)
4	Every two years	17(20.2)

5	As and when major changes are bought in curriculum/syllabus/TLM	26 (30.9)
6	Any other (Every Five Years)	19(22.6)

Regarding the frequency of organisation of in-service programmes, about one-third of the teachers responded that (30.9%), this should be organised as and when major changes are brought in Curriculum, Syllabus or Teaching Learning Materials (TLM). Around 22.6% expressed their interest of participating in in-service programmes every five years and 20.2 % opined that, the professional development programme needs to be organised every two years.

Lessons Learned

The implications from this survey report are very direct. Teachers in primary schools are in urgent need of Mathematics pedagogical improvement if they are to be expected to teach Mathematics effectively and they are very much aware of their own needs in this respect. The classroom practices used by the teachers reported through this survey is an indicator for the importance of organising practical based in-service programmes to the teachers in various pedagogical strategies. It is clearly evident that there is a need for teachers to be trained in more student-centred, activity-based learning methods which are appropriate to the primary school level.

The training programmes to be organised for the teachers also should consider exemplary materials on integrating student assessment with content and pedagogy. Various examples needs to be provided in the material and during the programme teachers needs to get opportunities to experiment in the real classroom. The training programmes needs to include practical sessions on engaging inclusive classroom as well as using ICT in teaching learning process.

The modality of the training also needs to be taken care off. Most of the teachers are interested in short term face to face programmes as well as blended programme. Longer duration face to face programmes need to be avoided and blended or face to face short duration (Five days) programmes may be planned instead of that.

The survey also helped in finalising the topics to be included in the training package apart from the framework of the package and modalities of the training. Care needs to be taken to develop the package in such a way that the content will be explained with the help of appropriate pedagogical strategy. The package should also give guidelines for assessing student performance through various formative assessment strategies.

In nut shell the Need Assessment Survey has given me enough confidence and motivation for developing the training package for primary Mathematics teachers.

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What is GeoGebra?

Abstract

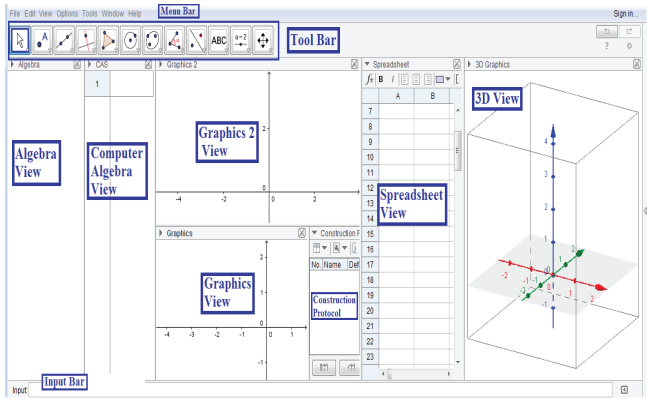
This paper is about the use of technology for explaining Mathematical ideas return in a simple structural manner this addressed these who want to began such explanation through Geogebra.

According to the website www.geogebra.org, "GeoGebra is dynamic Mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra is a rapidly expanding community of millions of users located in just about every country. GeoGebra supports Science, Technology, Engineering and Mathematics (STEM) education and innovations in teaching and learning worldwide."
It lists the Quick Facts as:

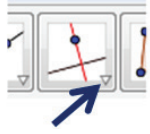
- Geometry, Algebra and Spreadsheet are connected and fully dynamic
- Easy-to-use interface, yet many powerful features
- Authoring tool to create interactive learning materials as web pages
- Available in many languages for millions of users around the world
- Open source software freely available for non-commercial users

GeoGebra 5.0 Interface

GeoGebra provides seven different views of mathematical objects as shown in the figure below. By default, GeoGebra displays Algebra view, Graphics view and Input bar when it is opened. The algebra view and graphics view allow us to display mathematical objects in two different representations: graphically (e.g. points, graphs) and algebraically (e.g. coordinates, equations). These two representations of the same object are linked dynamically and adapt automatically to changes made to any of the representations, no matter how they were initially created.

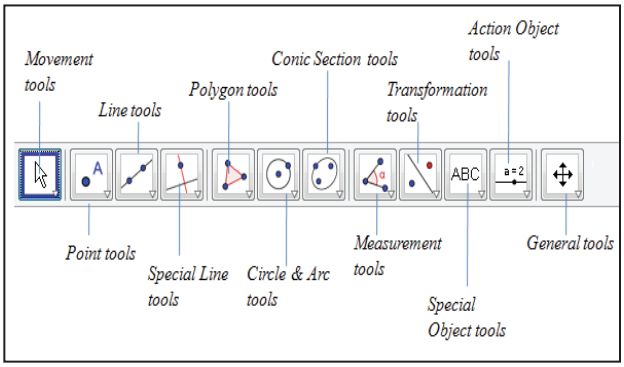


The toolbar at the top of the GeoGebra window consists of a series of toolboxes containing a collection of related tools. These tools can be used to create constructions in the Graphics View. Each View has its own Toolbar and therefore, gives access to a different set of tools. In the tool bar the active tool is highlighted by a blue border, see tool with arrow mark in above diagram. One can make a tool active by clicking on it.



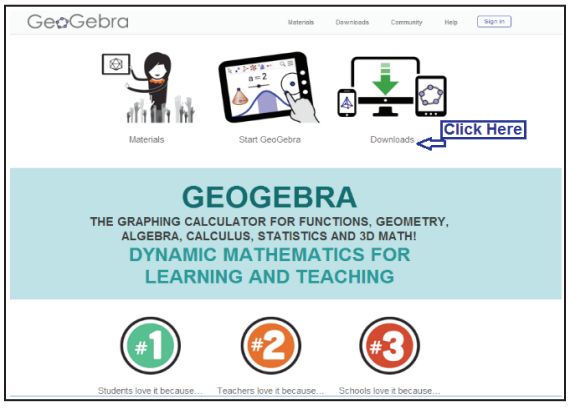
At the bottom right corner of each tool there is an inverted arrow, user can click that arrow to see similar tools in that group (Drop down menu)

The Input Bar at the bottom of the GeoGebra window is used to directly enter coordinates, equations, commands, or functions.



How to get GeoGebra

GeoGebra can be downloaded from the website www.geogebra.org. GeoGebra is available for various operating systems and can be used on tablets and android phones. All the downloads of GeoGebra can be obtained from the download page of the website. The web-start version of GeoGebra is also available on the home page. Otherwise one start an online Geo Gebra session without downloading and installing.



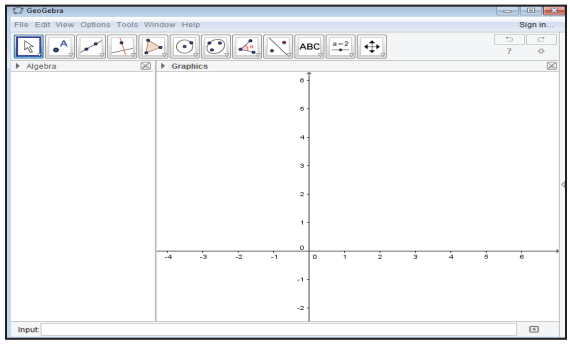
(GeoGebra Home Page)

(GeoGebra Download Page)

GeoGebra also have a large collection of material contributed from teachers around the world. It is maintained at tube.geogebra.org or www.geogebra.org/material . One can easily access these materials and contribute own material for the benefits of the others.

How to use GeoGebra

After the successful installation of GeoGebra on computer one gets the following icon on computer. To start the application double click on the icon then following launch screen with algebra view, graphics view, tool bar, input bar and menu bar will be presented.



Use of some of the tools presented

The Point Tool allows to put a point anywhere in the graphics view.

The Intersect tool allows to select two different curves and find their point of Intersection.

The Midpoint or Center tool is used to construct the midpoint of a line segment or to locate the center of a circle.

The Line tool draws an infinite line through two selected points.

The Segment tool is used to draw a line segment between two selected points.


The Perpendicular Line tool draws a perpendicular line to a given line.


The Parallel Line tool draws a line parallel to a previously constructed line.


The Polygon tool is used to construct a closed polygon on graphics view.

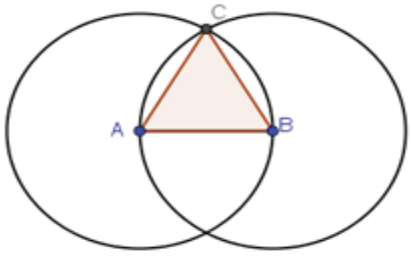

The Circle with Center through Point works by either clicking on the point one want to be the center of the circle or clicking on some blank space to create such a point. Release the mouse button. Move the mouse and you will see a circle in the process of construction. When click the mouse again, the circle is finished.


The Angle tool is used to construct and measure the angle between three points.

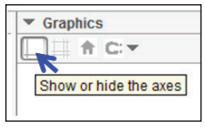
Let us see how to use some of the tools available in the toolbar by taking an activity of construction of an equilateral triangle.

Construction of Equilateral Triangle

In this construction we will use circle tool to demonstrate the work we do using straight edge and compass of Geometry Box. The idea is to use the two centers and the intersections of two circles to form a triangle as shown below.

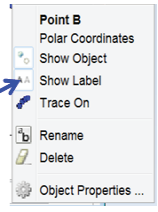


Here we will construct a triangle, display interior angles and length of segments. For the purpose of our construction, we don't need Algebra Window and Coordinate Axes, so we will hide them. To hide the *Algebra window*, click *View* then click *Algebra window*. To hide the *Coordinate axes*, you can click on the 'Show or Hide the Axes' key under the toggle Graphics Style Bar.

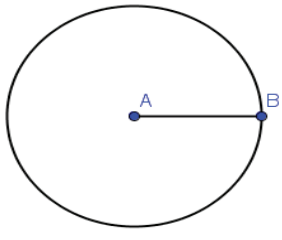


Now click the Segment between two points tool  , and click two distinct points on the graphics view to construct a segment AB.

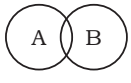
If the new points are not labeled then click on Move Button tool  then right click on each point and select Show Label from the context menu. (The *context menu* is the pop-up menu that appears when right click an object.) A and B should appear as label for two points.



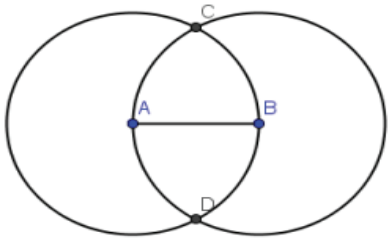
Now we will construct circle with center A and passing through B using the Circle with Centre through point tool  . Select that tool first and click point A and then click point B. After this step, the drawing should look like as shown below:



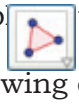
With same tool, now construct another circle with center B passing through A, click first on point B and then on point A.

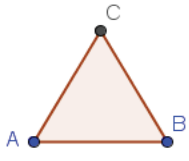


Next we will locate points of intersection of two circles. We will locate these points using Intersect two Objects tool  . Select the tool and click anywhere on the circumference of two circles. You will see that two points will appear at the intersection of two circles. Label them as explained above. After this step, the drawing should look like as shown below:



We need only three non-collinear (not in a line) points to form a triangle. Now hide the two circles, segment AB and point D. For this, right click on each object (circles, segment AB and point D) and select the toggle Show Object option from the context menu. Make sure not to click on points A or B.

Now, with only three points on graphics view , select (click) Polygon tool  and click the points in the following order : Point A, Point B, Point C and then Point A to close the polygon. After this step, the drawing should look like as shown below:

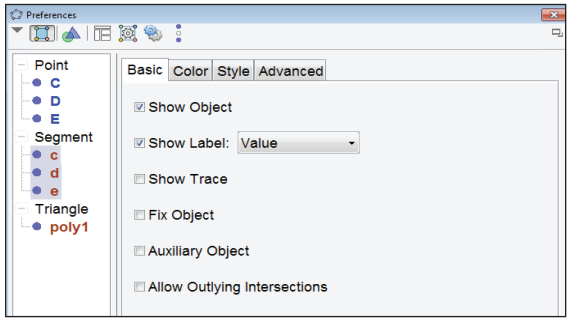


Next, using the Move Tool  try to move the vertices of triangle ABC. What do you notice, you will be able to move vertices A and B but not C. This is because vertex C is a dependent object, it is the intersection of two circles and thus depends on the length of segment AB.

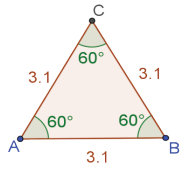
In the next stage, we have to verify that triangle ABC is an equilateral triangle. Note that a triangle is equilateral if all its interior angles are equal (60° each) or all its sides are of same length. Let us verify the interior angles.

Click on Angle Tool  and click anywhere inside the polygon (Triangle ABC). Note the measurement of interior angles.

Now let us verify the lengths of sides of triangle AB , BC and CA. This can be achieved using Property window. Right click on any of the sides, select Object Properties from the context menu. In the Object Properties window, select the Basic tab. Check Show Label box and choose Value from the Show Label drop down list.



Select the other two sides of the triangle under Segment section of Object list located at the left side of Object Properties window and change the Show Label to Value. Close the window when you are done. After this step, the drawing should look like as shown below: (You may have other values of length of sides of triangle.) Now try to drag points A or B , you will see that for every position of points A and B , the interior angles always remains 60° and the length of three sides are always equal. This proves that the above steps always results in construction of an equilateral triangle.



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From Kothari Commission to Contemporary System of School Education

Challenges in Attaining Parental Involvement in Children's Education

Abstract

The parent-teacher relationship has emerged as a topic of deliberation in the contemporary scholarship on education in India. Although latest policy documents indicate parental involvement as a necessary strategy for educational development, the inclusion of parents in schools' affair is relatively recent development in education practices. The delay in perceiving parents as a crucial participant in schooling experience of a child makes it imperative to get to the root of education planning and its development. This paper, based on a discourse analysis of Kothari commission's report, reviews how the home-school relationship or parent-teacher interaction were construed in one of the foundation documents in the history of modern education in India. Through attempting to understand commission's views on the role of parents in children's schooling and relating those perspectives with prevalent and contemporary education practices of education, the paper argues that there is a dire need for creating space for parents in order to achieve active engagement of parents in children's schooling experience.

Background

The contemporary Education system in India is characterized by unequal access and diverse schooling experience across variables of social stratification, such as gender, caste, class and so on. Amidst the evident diversity within and across various types of the schooling system within same education context, the emphasis of research in Social Sciences, and particularly in the field of sociology of Education, has been on the increasing, and arguably irreconcilable, inequality in the processes and functioning of the school system. Evidence provided in these studies suggests continued insufficiency and poor quality of infrastructure and services available to a large proportion of pupils (Ramachandran & Sharma, 2009). Although parents' involvement is recognised as crucial, the scholarly engagement in this topic remains largely obscured.

Amidst the persisting concern of building quality infrastructure and managerial issues for mass education, recent education policies acknowledge parental involvement in their children's education as irreplaceable and crucial strategy for educational development. However, this shift in educational planning has taken place relatively recently. It is curious, in this context, how modern education for independent India was envisaged, with specific reference to parents' engagement in schooling, by the policy makers of education. As this report laid the foundation for educational planning and development by providing a

comprehensive assessment of all levels of education in independent India, the text of analysis in this exercise is the report of Kothari Commission. I hope that this analysis would assist in making sense of contemporary challenges to attaining active participation of parents in their children's education.

This Paper is structured into three sections. The first section provides a brief background to the constitution of Kothari Commission and its primary contribution to envisioning the future system of school education in India. The second section reviews commission's views on parents and their positioning in education landscape. The third section provides commission's understanding of occasions of parent-teacher interaction and parental role within schools and schooling processes. The final section links the perspective of the commission to the contemporary education policies of Indian schooling system. The final remark is a snapshot of key observations of the paper and states the need for devising mechanisms for ensuring effective involvement and active engagement of parents in children's schooling.

Kothari Commission: an Introduction

Kothari Commission was not the first commission set up for discussing the educational planning of the newly independent India. Previously two commissions—University Education Commission (1948-49) and the Secondary Education Commission (1952-53)—were formulated to discuss the possible trajectory of educational development for the newly formed nation-state. On the 14th of July 1964, the government of India appointed a commission comprising seventeen members under the chairmanship of Professor D. S. Kothari—a trained physicist and Chairman of University Grant Commission—with a mandate "to advise the Government on the national pattern of education and on the general principles and policies for the development of education at all stages and in all aspects" (Government of India [henceforth, GoI], 1966, p. vii). The members of the commission were renowned academicians from India, United Kingdom, France, United States of America, Japan, the Union of Soviet Socialist Republics, and the representatives of United Nations.

The report was a result of two years of discussion and deliberations. It was titled "Education and National Development: Report of the Education Commission 1964-66", through which the Commission claims to have provided a "comprehensive review of the educational system with a view to initiating a fresh and more determined effort at educational reconstruction" (GoI, 1966, p. xii). Amidst varied issues such as economic deprivation, poverty, insufficiency of food supply, and unemployment, the commission viewed education as an "instrument of change" (GoI, 1966, p.6) that needed reforming such that it meets the requirement of developing country, i.e. to "increase productivity, develop social and national unity, consolidate democracy, modernize the country and develop social, moral and spiritual values" (note in Lall, 2005, p. 2). The commission's views are widely noted for advocating primacy to Science-based curriculum, the establishment of the common education system and suggesting three-language formulae.

Similar to other education commissions, KOTHARI COMMISSION reaffirms the Nehruvian idea of development through the implementation of the Science-based curriculum. The commission suggests nurturing academic talent through establishing and promoting agricultural

and industrial development based education. Science-based education, the commission views, "provide(s) the foundation as also the instrument for the nation's progress, security and welfare" (GoI, 1966, p. iii). In the culturally diversified and deeply socially stratified nation-state, the commission recommends the establishment of the common education system. This proposition implies that despite inherent hierarchies, education should be the same to all and equally accessible to everyone, especially to females, tribals, persons with disability, and to the socio-economically disadvantaged group (GoI, 1966, p. xiii-xiv). And finally, the three language formulae suggests the provisioning of learning a modern Indian language preferably one of the southern languages apart from Hindi and English in the Hindi-speaking States and Hindi along with the regional language and English in the non-Hindi speaking States.

Apparently, the above suggestions, along with many others, envisaged a sea change in not only in the system and contents of learning, but also in society in general. These changes would not have been possible without tremendous support from and plan on the part of the community in general and the parents of school going children in particular. In the following sectional, I try understanding how the commission viewed community and parents and their role in education planning and development.

Community and the parents: their roles and involvement

There is an uncanny resemblance between the critique of Kothari Commission and subsequent educational policies on the participation of community in education. While the commission discusses the importance of community in the building of a new nation, it recommends the provisioning of involvement of the community should be through "donations and contributions voluntarily made by the parents and local community from time to time" (GoI, 1966, p. 465). These funds, along with regular aid, were advised to be used for the maintenance of school property, school park, midday meal, purchasing of prizes, uniform, books and so on (see, p. 939). Hence, though the role of community was to provide requisite resources to school authorities from time to time, their presence remain external to school affairs.

Despite multiple forms of stratification based on the social positioning of the family—class, caste, religion, and so on—the commission categorizes parents into two overarching and simplistic categories: "privilege" and "average". KOTHARI COMMISSION maintains that British rule left India with an unequal system of modern education, which is supported by "privileged parents" in independent India. The British private schools functioned as a token of imperialism and worked with the specific curriculum for the children belonging to elite classes. Post-independence, these private schools were affordable to only a small proportion of Indian parents, whereas public schools catered to a vast majority of school-going children. The commission views fee of attending private schools as "anti-egalitarian" and "regressive form of taxation" (GoI, 1966, p. 186) and criticizes "privileged parents" for being gatekeepers of the class-based education system.

> …the economically *privileged parents* are able to 'buy' good education for their children…by segregating their children, such privileged parents prevent them from sharing the life and experiences of the children of the poor and coming into contact

with the realities of life (GoI, 1966, p. 15, emphasis added).

Thus, the Commission declares that privileged parents render the education of their children "anemic and incomplete" and weaken the "social cohesion" by meeting their class inspired aspirations through widening the gulf "between the classes and the masses" (GoI, 1966, p.14). On the other hand, kothari Commission does not hold a favorable opinion of the "average parents". The report condemns a majority of Indian parents for being apathetic towards their children's schooling. While explaining the reasons for unmet requirement of Article 45, the Commission upholds that the progress in educational development has been dismal, primarily because of "lack of adequate resources, tremendous increase in population, resistance to the education of girls, large numbers of children of the backward classes, general poverty of the people and the illiteracy and apathy of parents" (GoI, 1966, p. 298). While indicating parents' inability to send their children to school and a need for altering parents' attitude about education, the commission fails to suggest mechanisms to include both the categories of parents into the system of education.

As a remedy to the disparate system of education, the report recommends the establishment of Common School system (CSS). The concept of CSS, inspired by the schooling system in USSR, entailed availability and accessibility to free of cost education to all, irrespective of their positioning in social stratification. The goal of the education system, as envisioned by the Commission, is to maintain "adequate level of quality and efficiency so that no parent would ordinarily feel any need to send his child to the institutions outside the system, such as independent or unrecognized schools" (GoI, 1966, p. 463). This system, the commission argues, would cater to the need of average parent for their children may avail quality of education without having them spend a fortune on children's schooling.

The report maintains that "Gross inequalities (to avail educational opportunity) arise from differences in home environments", for instance," a child from a rural household or an urban slum having non-literate parents, does not have the same opportunity which a child from an upper-class home with highly educated parents has" (GoI, 1966, p. 181-182). The suggestion to this "problem" provided in the report is the "general improvement in the standard of living of the population" (ibid, p. 182). Hence, the commission blames the rural mass and urban slum dwellers, being backward and ignorant, for failing to create ambience for studying at home. The Commission does not allude to the home environment in its specificity, i.e. whether it is about the academic support, household arrangement, locality of stay, provisioning of resources, or general engagement with the teachers? Furthermore, the Commission does not recommend any forms of parental engagement with the education system and, therefore, excludes the parents—the key decision makers—from the system of education.

Summarily, without delving into the categorical specificities of the middle class and economically poor parents and devising any particular mechanism to include them in the everydayness of schooling experience, the Commission seem to instruct the parents what to do and what not to do. Hence, the Commission approaches parents rather condescendingly rather than providing them the status of a participant. While the parents and their categories are not rigorously defined and identified,

the commission clearly puts the efforts made by parents and the teachers in two different areas. Parents were suggested to build conducive home environment and take responsibility for sending their children to school and submit to the newly developed mass system of schooling. The report does not acknowledge that the community must be engaged in the process of creating such revolutionary form of learning in the modern system of school education.

Teachers and their role in building home-school relationship

Parents, as appears quite evidently in the previous section, were viewed largely as a recipient, and the Commission views with somewhat superiority. Normative supremacy of the teachers was "mainly framed under a foreign regime when control of the political views of teachers was a major objective of official policy" (GoI, 1966, p. 97). The report suggests the need "to frame separate and new conduct and discipline rules for teachers in government service, which would ensure them the freedom required for professional efficiency and advancement" (ibid). This efficiency, the commission suggests, is not only limited to the four walls of classroom teaching and learning, but also requires an irreplaceable and precious efforts for motivating parents to send their children to schools.

While suggesting the improvement in the infrastructural facilities, the Commission pointed out that since not every school has the infrastructure available for the teachers to live in the area where they teach, "proper" contact with the parents is not developed (GoI, 1966, p. 98). Since the relationship between school and home is largely determined and practiced through the communication between parents and teachers within schooling hours, living in the same place was requires in order to develop that connection.

Commission limits the function parent-teacher communication to improving attendance rate in school, i.e. "simple act such as a sympathetic enquiry made by a teacher of the parents whenever a child ceases to attend school" (GoI, 1966, p. 309) may enhance the rate of attendance and motivation among children and parents towards going to school. Another occasion of stressing parent-teacher relationship was in the case of "under-achievers" (GoI, 1966, p. 444), i.e. for the children who do not perform very well academically. Commission suggested, "parent-teacher associations should be mobilized for enlisting the cooperation of parents in dealing with special case" (GoI, 1966, p. 444, also see, p. 457). The relationship between parents and teachers was not recognized by the commission as a tool for attaining social cohesion between the two actors as a mechanism for maintaining teachers' accountability in the school. Rather, the inclusion of parents in the education system was deemed educative to the parents. Besides, the practice of parental involvement in schools was not discussed, leaving the scope of parents' participation ambiguous and unclear.

Commission suggests that parents "should be helped in the selection of courses for further education" (ibid, p. 438), but does not indicate any manner of informing the parents or the process of consulting with them the possible career paths for their children. The commission views families and parents as a hindering factor in the development of talented children and argues, "in a large majority of the homes, the environment is, deprivatory on account of the illiteracy of the parents and poverty, and does not allow the

available native talent to develop itself fully" (ibid, p. 440).

The inclusion of tribes in the formal education system was considered a challenge and the suggestion of the commission is the "intensive programme of parental education" (ibid, p. 228). Similarly, along with the proposal for conducting training for the teachers in pre-primary schools, the commission suggests to "provide education to parents regarding child care" (ibid, p. 292). Besides blaming the parents and their non-progressive ideas about education and development, lack of adequate infrastructure was considered as a fundamental problem in meeting universalization of primary education. This includes,

> (...) existence of incomplete schools which do not teach the full courses; the large prevalence of stagnation which discourages children from staying longer at school; the dull character of most of the schools and their poor capacity to attract students and retain them; the absence of ancillary services like school meals and school health (ibid, p. 308)

Other factors on the part of parents such as, "reluctance of parents either to educate their daughters further or to send them to mixed higher primary schools" (ibid p. 299), and "failure of the average parent or child to see the advantage of attendance at school" (ibid, p. 308) were also considered equally crucial. While developing infrastructures and improving teachers' attendance were recommended, the commission proposed "an intensive programme of parental education" (ibid, p. 308, 343) with an objective to "persuading the parents to accept the inevitability of mixed schools for boys and girls" (ibid, p. 299). In other words, the efforts were directed to "convince" (ibid, p. 420) parents, instead of discussing with them, that the changes made in the education system were inevitable and should be welcomed.

Contemporary relevance of Kothari Commission's report and challenges ahead: a discussion

Many observations in Kothari Commission's report are relevant to the scenario of contemporary education. This section summarizes the key observations made in this paper and how those are still pertinent to the contemporary discussion of the home-school relationship.

It's imperative to understand and acknowledge the institutional difference in existing hierarchical social order of Indian society and relatively egalitarian establishments of the common education system. Failure to assess these differences resulted in a ***variously stratified***, more complex system of contemporary education in India. Put differently, democratic thoughts of Kothari Commission were indeed welcoming. However, the commission did not think through the ways and processes of bridging the gap between the hierarchically stratified society of India and principles of common education system.

The Commission views two ***categories of parents***: one who are wealthy and other who are poor. Such simplistic division fails to capture the positionality of individuals amidst the prevailing intersectionality posited through the interplay of class, caste, religion, region, language, and so on. Disadvantaged in India is not a homogeneous group; it varies across states--one community which is in the minority in one state may be a majority and dominant in the other. In the report, while privileged parents were marked as consumers, socio-economically deprived—especially urban poor, rural, tribal parents—were seen as backward, apathetic, passive, beneficiaries of the education.

Later, with policy interventions, various platforms such as Village Education Committee, School Betterment Committee, and Parent Teacher Association were provided to the community members and exclusively to the parents for observing and becoming a part of aspired education development. Similar to the issue with Kothari Commission, the failure in attaining expected goals for parental involvement in education lies in defining "Community" as a simplistic category with homogenous groups. The suggested, "complex, diverse, dynamic, and the mythical notion of community cohesion actually glosses over differences and divisions while privileging the voices of people who have more power" (Guijit & Kaul Shal, 1998, mentioned in Saihjee, 2004, p. 231). Also, the stronger emphasis on the community without specifying the participation of parents is problematic, because "strengthening mechanisms for community participation without ensuring the participation of parents is often counterproductive" (Ramchandran, 2004, p. 84) for attaining focused relationship between teachers and parents.

Furthermore, although Kothari Commission mentions the role and importance of ***Community participation***, it remains largely exclusionary to the processes of schooling. Also, the suggested ways of participating in the schooling process--through providing resources to the school--is possible to the socially and economically affluent families. Following up on the same principle, the contemporary practice of representation of the dominant group in school management committee in school processes reproduces the power structures of society. Such practices also hinder the correcting mechanism when children from disadvantaged families are subjected to social biases, discrimination, and negligence by the teachers at school. Probe (2011) notes the discrimination based on the caste of pupils and its repercussion on a parental decision about selection of schools. The Probe team observes that since most of the teachers in government schools were upper caste, neglect of the children from Schedule Caste (ibid p. 64) turned out to be the primary reason of selecting the low-fee private schools, even though parents could barely afford the costs of education. In contexts where parental participation has transcended the structural barriers, trends of increasing enrolment; reducing dropout rate has been noted (Rathnam, 2004).

Although recent education policies tend to focus on community participation, community per se is not viewed as a stakeholder in making decisions regarding either school policies or managing everydayness of schooling arrangements (see Govinda & Bandyopadhyay, 2011; Govinda, 2002). Since "primary education as the invention is bound by the predestined purpose" (Kumar, Priyam, & Saxena, 2001), communities do not have a space to voice their opinions. With the limited scope of participation in the schooling system, it is assumed that the importance of education would be realised from within the community. This assumption does not necessarily imply, for instance, that "community contributes towards a reasonable space for the school and identify a suitable teacher" (Ghosh, 2004, p. 129).

Instead of suggesting ways to overcome class and caste barriers and develop a functional relationship between home and schools, the commission blames "average parents" for not sending their children to schools and keeping the talented ones away from the mainstream society--KOTHARI

COMMISSION equates illiteracy with uneducatedness. Today, socially and economically disadvantaged parents struggle in deciding between schooling and employment for their children (Jain, Mathur, Rajgopal & Shah, 2002). Among working class in rural areas, parental involvement in everyday schoolings—through participating in the parent-teacher association, volunteering, or helping with homework—is rather limited, but parents are the key decision makers in school-related decisions such as selection of schools, a continuation of education for sons and daughters, and so on (Maertens, 2011). Limited ways of parental engagement fail to acknowledge that for the majority of people living below poverty line in India, attitude of the parents towards schooling is the primary "driver of children's educational outcomes" (Probe, 1999, p. 45).

The modern education, which British introduced in India, did not see Indian parents as perhaps useful resources within the Western framework of teaching and learning. KOTHARI COMMISSION reinforces this ideology to the proposed education practices through accommodating parents only in a capacity to provide the suitable home environment. Hence, through reinforcing colonial practices of social sanction, KOTHARI COMMISSION's recommendations tend to disjoint and widen the area of work between parents and the schools. While the Commission acknowledges the difference between teachers' role in independent India, as compared to their responsibilities during colonialism, indicating that teachers are no more the servant of the government, rather they should build the network within the community. The ways of bridging the gap between the teachers and parents were not duly brainstormed or sufficiently laid out. Its noteworthy that while the commission does not discuss the parental role in the contribution to making the new system of education, it maintains that schools and schoolteachers are solely responsible for educating the child.

The Kothari Commission notes that with the provisioning of the common education system, "the average parent would not ordinarily feel the need to send his children to expensive schools outside the system" (GoI, 1966, p.15). Though mass education was introduced in India, the government schools did not work sufficiently well and failed to provide quality of education, simultaneously, the number of private schools that offered competitive fee structures increased in number. Generally, across different regionals in the country, families, including socially and economically disadvantaged, "often prefer to incur additional expenditure and send their children to private rather than government schools" (Sedwal & Kamat, 2011, p. 105; also see Probe, 2011).

Final remarks

Division of these two institutions—family and school—and two stakeholders—parents and teachers—result from the lack of mutual effort in the construction and sustainment of education. In other words, the discussion on the parent-teacher relationship does not extend to the level of active participation of the parents; it also implies that parents need not to be actively participating in the educational attainment of their children. This conceptualization of home-school relationship did not give a chance to generating a discussion at the local level about the establishment of schools and the need for education.

Hence, schools were perceived not as a local institution but an outside body regulated by the state.

The only two occasions at which the teachers and parents were suggested to get in touch with each other were absenteeism of teachers and discussing the causes of under-achievement of the student. Hence, education appears to be an imposition of state's ideology onto the people, especially to the rural inhabitants and tribes who were never a mainstream concern of the British and, therefore, were less acquitted with the 'modern education system'. The approach to incorporate parents into educational institution was largely top-to-bottom, quite opposite to the ideological establishment of the democratic nation state. In today's context, in light of the above discussion, it is imperative for teachers to step forward and make space for parents to share their concerns and issues so that the education system would have an added, and perhaps more effective, form of governance that would monitor the quality of education closely and effectively.

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तोत्तो-चान

किताब की समीक्षा

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तोत्सुको कुरोयानागी ने यह किताब अपने स्कूल के संस्थापक श्री सोसाकु कोबायाशी को समर्पित की, कि किस तरह तोत्तो-चान को एक सफल छात्रा बनने में उन्होंने उसकी और बाकी तोमोए स्कूल के छात्रों की मदद की। एक बच्चे के जीवन में एक शिक्षक का कितना महत्व होता है इसका अन्दाजा हम इस किताब को पढ़ने के बाद लगा सकते हैं। साथ ही यह भी कि ऐसे कितने लाखों बच्चे होंगे जिन्होंने अपनी पढ़ाई शायद इसलिए बीच में ही छोड़ दी होगी क्योंकि वे कक्षा में या स्कूल में शैतानियां करते थे और जो शिक्षकों को रास नही आता था इसके कारण उन्हें या तो स्कूल से निकाल दिया जाता होगा या उन्हें इतना डरा धमका के रखा जाता होगा कि वे स्वयं ही स्कूल छोड़ने को मजबूर हो जाते होंगे। एन सी एफ 2005 हो या शिक्षा नीति सभी में इस बात की पैरवी होती रही है, कि बच्चों को स्कूलों में भय मुक्त वातावरण दिया जाए। यह मेरी व्यक्तिगत राय है कि कागजों में तो यह शत प्रतिशत हुआ कि बच्चों को फेल नही करना है, डराना धमकाना नही है किन्तु धरातल पर शायद ही कुछ प्रतिशत हो पाया हो। अगर हम भारत के परिपेक्ष में देखें तो यह बात हमेशा किसी ना किसी लेख में छपी होती है कि बच्चों को खुला वातावरण दिया जाए, भय मुक्त शिक्षा होनी चाहिए। परन्तु आज भी हमारी शिक्षा व्यवस्था या स्कूलों में ऐसा माहौल नहीं आ पाया कि बच्चों को एक खुला वातावरण मिलें और वे वही कर पाऐं जो करने का उनका मन करता है। मेरा मानना है कि लगभग हर बच्चे के अन्दर एक तोत्तो चान छुपा होता है। आपने भी अकसर घर या स्कूल में बच्चे को एक ही काम को बार बार करते देखा होगा, कृष्ण कुमार जी ने भी अपनी किताब 'बच्चे की भाषा और अध्यापक' में यह कहा है कि बच्चा ऐसा इसलिए करता है कि एक तो उसको उसमे मज़ा आता है और दूसरा वह उस चीज को छू कर महसूस कर उसके बारे में अपने अनुभव पक्का करता है। बच्चे का बार-बार एक ही काम करना अध्यापक को तो क्या माता-पिता को भी पसन्द नही आता है। वे उसकी जिज्ञासा और रूचि पर पाबन्दी लगा देते हैं। तोत्सुकी कुरोयानागी ने इस किताब में यह भी लिखा है कि अगर सोसाकु कोबायाशी उसको यह मौका नहीं देते तो या तो वह स्कूल छोड़ चुकी होती और या तो उसके माथे पर एक 'खराब लड़की' का ठप्पा लग चुका होता, और वह आज कुठांओं से भरी एक भ्रमित महिला होती। सोसाकु कोबायाशी का बार-बार यह कहना कि "तुम सच में एक अच्छी बच्ची हो" ने उसे सबल बना दिया। वह कहती है कि अगर आज तोमोए जैसे ढ़ेरों स्कूल होते तो चारों तरफ व्याप्त हिंसा कम होती और इतने बच्चे स्कूलों से पलायन नहीं करते। तोमोए

में स्कूल की घ्ंटी खत्म होने के बाद भी कोई बच्चा घर लौटना नही चाहता था। और सुबह सबको स्कूल पहुँचने की उतावली होती थी।

स्टेशन और खिड़की में खड़ी नन्ही लड़की

तोमोए के स्कूल जाने से पहले माँ ने उसे यह नही बताया कि उसे स्कूल से निकाला गया था। जब तोत्तो-चान अपनी माँ के साथ स्टेशन से नये स्कूल जा रही थी और उसके मन में टिकट अपने पास रखने को लेकर जो विचार आया, उससे आप उसके पहली बार रेलगाड़ी में बैठने का अनुमान लगा सकते हैं कि वह कितनी उत्सुक थी। बिना किसी डर के उसने टिकट बाबू से टिकट अपने पास रखने की इच्छा जताई तथा टिकट न मिलने से उसने यह विचार बनाया कि वह रेलगाड़ी में टिकट बेचेगी। इस दौरान उसने जो बातचीत टिकट बाबू और अपनी माँ से की उससे यह अनुमान लगा सकते हैं कि हर बच्चे के मन में कुछ ना कुछ करने की इच्छा होती है और उसे कैसे वह अपने पूर्व ज्ञान से जोड़ता है जो उसे अच्छा लगता है। तभी तोत्तो-चान ने माँ से कहा कि मैं सोचती हूँ कि मैं नुक्कड़-साजिंदों की टोली में शामिल हो जाऊं, जो नई दुकानों में प्रचार करते हैं। इस बात से तो उसकी माँ को भी निराशा होने लगी थी। शायद इसलिए वे अपनी कूदती फांदती बच्ची के लिए ऐसा स्कूल ढूँढना चाहती थी, जहां उसकी नन्ही को वे समझें, जहां वे उसकी बच्ची को दूसरे बच्चों के साथ रहना-पढ़ना सिखा सकें।

उसके पुराने स्कूल से उसे केवल इसलिए निकाल दिया गया था क्योंकि वे कई बार अपनी मेज खोलती, साजिंदों से बात करती, दिन भर खिड़की पर खड़ी रह कर साजिंदों का इन्तज़ार करती और साथ ही साथ चिड़ियों से बात करती। इन सभी से तंग आकर शिक्षिका ने उसे स्कूल से निकाल दिया। तोमोए में उसे वे सारे मौके मिले जिसे वह करना चाहती थी। तोमोए उसे इसलिए भी पसन्द आया क्योंकि वहाँ उसके किसी भी काम को करने में रोक नही थी।

नया स्कूल- तोमोए गाकुएन

तोत्तो-चान को अपना नया स्कूल अपने पहले स्कूल से ज़्यादा पसन्द आया। यह स्कूल उसके पहले स्कूल से भिन्न था। स्कूल का गेट पेड़ के तनों से बना था। स्कूल के कमरों की जगह रेलगाड़ी के डिब्बे। तोत्तो-चान को लगा कि ऐसा तो सपने में ही होता है, रेलगाड़ी में स्कूल! वह खुशी से चिल्लाई और रेलगाड़ी के डिब्बों की तरफ भागी और उसने माँ से कहा कि "स्कूल मुझे अच्छा लगा"। यह पढ़कर मुझे भी महसूस हो रहा है कि काश ऐसा स्कूल हर बच्चे को मिले। तोत्तो चान ने स्कूल में पहुँचते ही हेडमास्टर की तुलना स्टेशन मास्टर से की क्योंकि उसने सोचा कि अगर वे इन सब रेलगाड़ियों के मालिक हैं तो वे स्टेशन मास्टर हुऐ ना।

हेडमास्टर से मिलते ही उसने उनसे सवाल किया कि आप हेडमास्टर है या स्टेशन मास्टर? और ऐसे ही वह हेडमास्टर के साथ चार घण्टे तक बतियाती रही। हेडमास्टर को भी उसे सुनना उतना ही अच्छा लगा जितना उसे अपने बारे में बताना। जब वह अपनी सारी बात पूरी कर चुकी तो उसे यही सुनने को मिला कि तुम अब इस स्कूल की छात्रा हो। फिर तोत्तो-चान को उन्होंने उसका स्कूल और दोपहर के भोजन का स्थान दिखाया जहां उन्होंने बच्चों से पूछा कि कुछ पहाड़ से और कुछ समुद्र से लाए हैं कि नही। बच्चे किलकारिया मारते हुये अपने अपने डिब्बों को दिखा रहे थे। तोत्तो-चान के मन में यह विचार आया कि कल वो भी बच्चों के साथ बैठकर भोजन कर रही होगी वह मन ही मन सोचती रही कि उसका यह स्कूल पहले से कितना अलग हैं। और वो बेसब्री से अगले दिन का इन्तजार करने लगी। नये स्कूल की कक्षायें ट्रेन के डिब्बों में लगती थी। पिछले स्कूल में बच्चों के बैठने के स्थान निश्चित थे किन्तु यहाँ कोई भी कही भी बैठ सकता था। तोमोए में पढ़ने की कोई घण्टी नही होती थी। शिक्षिका ने पूरे दिन भर में पढ़ाऐ जाने वाले प्रश्नों की सूची बनायी होती थी और बच्चों को जहाँ से शुरू करना हो कर सकते थे। पढ़ने-पढ़ाने की इस पद्धति से शिक्षक हर बच्चे पर नजर रख सकते थे; उनकी रूचियों, उनके विचारों, उनके चरित्र से बखूबी परिचित हो सकते थे। अपने छात्रों को गहराई से जानने का यह आदर्श तरीका था। विद्यार्थी अपने चहेते विषय से दिन की शुरूआत कर सकते थे। तोमोए में जैसे

भोजन को आनन्द से खाया जाता है, कक्षा में शिक्षण भी उसी प्रकार किया जाता है। बच्चों का मन अगर सैर करने का है तो वे सैर पर ही जायेंगे पढ़ने में कोई दबाव नहीं।

तोमोए की हर चीज बिल्कुल अलग थी चाहे वह कक्षाओं का संचालन हो या पढ़ने पढ़ाने का माहौल बच्चे अपनी इच्छा के अनुसार काम करते। बच्चों को आपस में बातचीत के भरपूर मौके दिये जाते ताकि वे एक दूसरे को जान सके।

समुद्री खाना-पहाड़ी खाना

हेडमास्टर जी ने यह जुमला संतुलित आहार के लिए बनाया था ताकि हर बच्चे को पौष्टिक खाना मिल सके। माता-पिता को यह समझाने के बजाय कि बच्चों को पौष्टिक खाना दें वे कुछ समुद्र से कुछ पहाड़ से लाने को कहते। बच्चों के मन में भी हमेंशा यह शंका रहती कि वे पहाड़ से और समुद्र से की शर्त को पूरा कर पाते हैं या नही। साथ ही यह जानना भी रोमांचक था कि समुद्री आहार और धरती का आहार क्या होता है। साथ ही बच्चों को खाने में यह उत्सुकता रहती कि कौन बच्चा क्या लाया है। खाने के दौरान यह कहना कि ठीक से चबाकर खाओं कहने से अच्छा उन्होंने एक गीत ही बना लिया था:

चबाओ, चबाओ, ठीक से चबाओ,
जो कुछ भी तुम खाओ,
चबाओ, चबाओ, ठीक से चबाओ,
चावल, मछली, मांस!

साथ ही बच्चों को हमेशा यह राय दी जाती कि वे आराम से भोजन का आनन्द लेकर खाऐं।

सैर

शिक्षिका की बनाई सूचि से बच्चों ने अगर सारे काम ठीक से किए हैं तो ही उन्हें सैर पर जाने को मिलता इस बात से बच्चे परिचित थे इसलिए सारे बच्चे अपने पूरे दिन का काम पूरे मन से करते थे। यह बात सभी कक्षाओं के बच्चों पर लागू होती थी। सैर पर जाते और खेलते कूदते बच्चों को यह कत्तई अहसास नही हुआ कि खेलने-कूदने और आजादी का समय असल में विज्ञान, इतिहास और जीव विज्ञान के पाठ थे।

रेल के नये डिब्बे का आगमन

स्कूल में नये डब्बे का आगमन जैसे ही बच्चों ने सुना उनके मन में उत्सुकता जाग उठी। बच्चे तरह-तरह के अनुमान लगाने लगे कि रेल का डिब्बा स्कूल तक कैसे आयेगा क्योंकि उस समय आज की तरह बड़ी क्रेनें तो होती नही थी। यहाँ तक कि बच्चों ने रेल के डिब्बे के आगमन के लिए रात को स्कूल में रूकने का आग्रह तक कर लिया था।

तोमोए स्कूल में बच्चों के लिए तरह तरह के नये अनुभव मिले चाहे वह बहादुरी की परीक्षा हो, तरण-ताल निर्वस्त्र नहाना हो, यात्रा गर्म सोते की हो, स्कूल में सबसे खराब कपड़े पहनकर आना हो या गर्मियों की छुटिया मनाना हो। बच्चों के निर्वस्त्र नहाने में हेडमास्टर का यह मानना था कि लड़के-लड़कियां एक दूसरे के शरीर को लेकर कुत्सित उत्कंठा मन में ना रखें और तोमाए में कुछ बच्चें ऐसे भी थे जो शारीरिक विकलांग थे। वे बच्चों को यह सिखाना चाहते थे कि हर शरीर का सौन्दर्य होता है। तोत्तो-चान ने जब यासुकी चान को अपने पेड़ पर चढ़ने का आमंत्रण दिया वह कितना जोखिम भरा काम था क्योंकि यासुकी को पोलियो था और वह पेड़ पर नही चढ़ पाता था। तोमोए में बच्चों को अपने मन मुताबिक काम करने की आजादी तो थी ही साथ ही उन्हें यह भी सिखाया जाता था कि उन्हें अपने सहपाठियों के साथ कैसे व्यहार करना होता है। बच्चों के लिए खेल भी ऐसे ही इजात किये गये थे कि हर बच्चा उसमें प्रतिभाग कर पाये। साथ ही यह काफी महत्वपूर्ण था कि बच्चों को इनाम में सब्जिया भेंट करना ताकि बच्चे यह सीख सकें कि वे भी घर में अपना सहयोग कर सकते हैं।

पुस्तकालय का डिब्बा

बच्चें जब सर्दी की छुट्टियों के बाद स्कूल आये तो उन्हें एक नयी चीज़ देखने को मिली। हेडमास्टर ने कहा यह है तुम्हारा पुस्तकालय जो चाहे किताब पढ़ सकते हो। बच्चे बहुत खुश हुऐ जिन्हें पढ़ना आता था उन्होंने पढ़ना शुरू किया और जिन्हें पढ़ना नही आता था वे भी किताबों को उल्ट-पलट कर देखने लगे। बच्चे जब भी बारिश या किसी कारण से सैर पर नही जा पाते वे पुस्तकालय में ही अपना समय बिताते।

खेती-बाड़ी के शिक्षक

बच्चों को खेती-बाड़ी के बारे में सिखाने के लिए हेडमास्टर ने यह तरीका निकाला था कि बच्चों को खेती के बारे में एक किसान से बेहतर कोई नही सिखा सकता है। बच्चों ने अपने खेती-बाड़ी के शिक्षक के साथ अपने हाथों से खेत में बीज बोया और उनके कहने पर बच्चों ने फावड़े, कुदालें चलाना सीखा। उन्होंने बच्चों को खरपतवार के बारे में जानकारी दी और साथ में कीड़े-मकोड़ों, तितलियों, चिड़ियों और मौसम की जानकारी भी दी।

तोमोए में काफी कुछ खुद महसूस करके सिखाया जाता था। जैसे खुले में रसोई पकाना व खाने को स्वयं पकाना और पकते हुये देखना। चाक का इस्तेमाल करना कि बच्चे रास्ते और दिवारों को गन्दा ना करें। तोमोए का स्कूल सच में एक अदभुत स्कूल था। जिसके बच्चे बड़े होने के बाद भी वही कर पाये जो वे करना चाहते थे।

मेरी राय- तोत्तो-चान के बारे में जितना कुछ इस किताब में लिखा है उसे पढ़ते समय मैं स्वयं महसूस कर पा रही थी कि अगर बच्चों को सही तरह से मार्गदर्शित किया जाए और समझा जाए तो हर एक बच्चा तोत्तो-चान बन सकता है। क्योंकि अधिकतर लोग बच्चे की गतिविधियों के अनुसार उन पर लेबल लगा देते हैं कि यह ऐसा बच्चा है वैसा है। तोमोए में बच्चों को अपने चहेते विषयों पर काम करने की छूट मिली और उनकी प्रतिभाओं में निखार आया। साथ ही शिक्षकों के प्रोत्साहित करने वाले शब्द बच्चों के जीवन को गढ़ने में निर्णायक घटक रहे हैं। हेडमास्टर का तोत्तो-चान को हमेशा यह कहना कि "तुम सचमुच अच्छी बच्ची हो" यही बात हर बच्चे के लिए महत्वपूर्ण होती है। तोमोए स्कूल का वातावरण तो अच्छा था ही साथ ही साथ तोत्तो-चान के माता-पिता का भी काफी योगदान रहा उन्होंने उसे कभी भी यह नही कहा कि उसने कुछ गलत किया यहां तक कि स्कूल से निकाले जाने पर भी तोत्तो-चान की मां ने उसे यह नही बताया कि उसे स्कूल से निकाला गया। अपने कक्षा शिक्षण के दौरान मेरा भी यही मानना था कि बच्चे अगर शान्त होकर सुनेंगे या पढ़ेंगे तो उन्हें ज़्यादा सीखने को मिलेगा। तोता चान पढ़ने के बाद मैं भी यह सोचने को मजबूर हूँ कि अगर बच्चें अपने नियम स्वयं बनाऐ तो वे ज़्यादा अधिक सीख पाऐंगे। बच्चें सचमुच प्यार के भूखे होते हैं। मेरी कक्षा में एक बच्चा है चन्दन, चन्दन को स्कूल आना पसन्द नहीं है ऐसा उसने बताया कि स्कूल में खेलने को नही मिलता है इसलिए वह स्कूल कम आता है। मेरे स्कूल में जाने के बाद हमने थोड़े बहुत बदलाव किऐ थे कि बच्चों का मन जो भी करने का है वही कार्य स्कूल में किया जाए। साथ ही खेल गतिविधियों के साथ पढ़ाई करना शुरू किया और बच्चों से एक-एक करके भी बात करना शुरू किया। कुछ सप्ताह में देखा तो चन्दन को स्कूल आने में मजा आने लगा और वह अनुपस्थित कम रहने लगा। ऐसे ही प्रत्येक बच्चे को मौका देते हैं बात करने का अपनी बात कहने का और दूसरे को समझने का तो कक्षा में काफी काम किया जा सकता है। हमें यह भी मानना होगा कि बच्चे अपने काम का संचालन करना बखूबी जानते है। जैसे तोत्तो-चान के कोई भी काम करने में चाहे वह हौदी में से बटुवा निकालना हो या किसी भी चीज़ के ऊपर कूद जाना हो और गन्दे होकर घर आने पर उसको बिना किसी डांट मार के समझाना ताकि वह अगली बार कोई भी काम करने से पहले सोचे जरूर। तोमोए स्कूल के शिक्षकों का बच्चों के लिए प्यार और शिक्षा के लिए समर्पण ही एक बहुत बड़ी बात थी। बच्चा जो भी चीज़ स्वयं करके सीखता है वह अध्यापक के बताने से कई गुना ज़्यादा सीखता है। यह बहुत बड़ी बात होती है बच्चों को समझाना कि सब बच्चे एक जैसे होते हैं तोत्तो-चान की मां ने उसे जब यह समझाया था कि तुम यह कभी मत सोचना कि लोग अलग-अलग होते हैं। अध्यापकों का बच्चों से जुड़ाव और साथ ही बच्चों का आपस में एक दूसरे के लिए सहानुभूति होना भी बहुत जरूरी है। अध्यापकों के साथ साथ बच्चे भी एक दूसरे की समस्याओं को समझ सके। जैसे तोत्तो-चान का मन साजिन्दों पर लग गया था वैसे ही किसी भी बच्चे का मन किसी भी चीज़ में लग सकता है, उसको समझना होगा।

Ganit Saptah

The Position Paper of the National Focus Group on Teaching of Mathematics developed by the NCERT during the exercise of the National Curriculum Framework-2005 says that *Mathematics offers a way of doing things and develops the quality of attacking all kinds of problems in a systematic manner*. This makes a learner confident in handling different complex situations in her daily life. However this can be achieved if the learners are exposed to different situations in Mathematics in which they could get an opportunity to explore and analyse for themselves. For this purpose the activity based learning and teaching in Mathematics would be useful. One of the modes through which this can be done apart from classroom Mathematics activities is, organizing different Mathematics exhibitions or fairs. In such events the learners get an opportunity to express their ideas before others fearlessly. This requires working on the related mathematical concepts and then expressing them.

NCERT has made an attempt to move towards fulfilling these objectives by proposing a weeklong event called GANIT SAPTAH, to be celebrated in all schools of the country. The activities on each of the days during this event could be for smaller durations so that students get enough time to interact with all and the schedule of regular school activities is also not affected much. The interactions and discussions on Mathematics and issues related to its learning will deepen the understanding of Mathematics in the students and teachers. This will also result in better dissemination of ideas and material. This week long activity is proposed to be conducted around the 'National Mathematics Day' which is celebrated in the memory of the great Mathematics legend Srinivas Ramanujan, on 22nd December.

Apart from students' activities, teachers are also expected to participate in this event by engaging in discussions on Mathematics and its transaction, among themselves. During the SAPTAH there will be discussions among students about Mathematics and their general observations about its learning; panel discussions of students/teachers; invited talks of experts/teachers; poster presentations/display of Mathematics exhibits by students; any other relevant activity that the school deems fit, etc. Students from classes I to XII are expected to participate in this event.

A letter has been sent by NCERT to all States and Union Territory requesting them to direct their schools for the celebration of this event.

National Conference On Mathematics Education at Regional Institute of Education (RIE), Bhubaneswar

(Report)

Mathematics education is a key to increase the post-school and citizenship opportunities of young people, but today, it is observed that many students struggle with Mathematics and become disaffected as they continually encounter obstacles to engagement with Mathematics. It is imperative, therefore, to understand what effective Mathematics teaching looks like—and what teachers can do to break this pattern. The teaching community is engaged in addressing the concerns related to the learning of Mathematics by the students, like, how teaching of Mathematics affects the learning attitude of Mathematics in the learners including learners with disabilities; *what* Mathematics should be learned and how; how to engage children in the meaningful learning of Mathematics; how Information and Communication Technology (ICT) can be used to improve transaction of Mathematics especially in large classroom sizes, etc. The practitioners researchers at different places try to innovate methods to make learning of Mathematics accessible to the learners and in turn gain some experience. These experiences need to be shared among the teachers, teacher educators and students for the benefit of the learners' community. NCERT has taken initiative to provide a platform to bring all such practitioners in the field of Mathematics together by organising National conference in Mathematics Education. This is held every year in one of the RIEs of NCERT in rotation on or around 22nd December, the birth anniversary of the great Mathematics legend S.Ramanujan. Experts, teachers, teacher educators, researchers and students in the field of Mathematics from all over the country participate in this event and make their presentations on the given themes and subthemes of the conference based on their first hand experiences in the field. This is accompanied by thought provoking discussions on the given deliberations.

In this series, the next conference was at RIE, Bhubaneswar on 19-20 December 2016. The themes for this year were:

- Professional Development of (pre-service and in-service) Mathematics Teachers
- ICT in Mathematics Education
- School Mathematics Curriculum
- Mathematics in nature and other disciplines
- Twenty-first century Mathematics learning-issues and challenges
- Assessment in school Mathematics
- Interventions in early school Mathematics
- Mathematics laboratory and other learning resources

टीचर लर्निंग सेंटर

(रिपोर्ट)

प्राइमरी मैथ्स और टी.एल.एम.

उत्तराखण्ड ने विकासखंडों की साझेदारी से टीचर लर्निंग सेंटर की तीसरी गतिविधि 24 शिक्षकों के साथ आरम्भ हुई। अधिकांश शिक्षक 3.20 तक पहुँच गये तो उनसे अनुमति लेकर सत्र की शुरूआत भी हो गई।

सुगमकर्ता नें अपना परिचय देते हुए गतिविधि का उददेश्य बताया और कहा कि हम इस दौरान कुछ चीजें आपसी बातचीत के माध्यम से साझा समझ बना कर करेंगे:

- टीचर लर्निंग मैटीरियल या लर्निंग रिसोर्सेज पर समझ बनाना।
- प्राइमरी स्तर पर टीचर लर्निंग मैटीरियल की आवश्यकता।
- विभिन्न तरीकों से टीचर लर्निंग मैटीरियल का प्रयोग व सीमांए।

सत्र का प्रारम्भ व्यक्तिगत नाम से न पूछकर एक गतिविधि से हुई। सभी प्रतिभागियों से स्वयं के जन्मदिन को जोड़ते हुए एक अंकीय संख्या में लाने को कहा गया। उसके बाद अपनी एक संख्या को लेते हुए एक बड़े गोल घेरे में खड़े होने को कहा गया। तत्पश्चात सुगमकर्ता द्वारा कुछ निर्देश दिए गए जिनके आधार पर दो-दो समूह बांटे गए व प्रतिभागी अपने साथियों से उनके नाम पूछते गए। इस गतिविधि के माध्यम से सभी ने एक दूसरे का नाम जाना।

- सम संख्या वाले एक तरफ व विषम संख्या वाले एक तरफ
- भाज्य संख्या वाले एक तरफ व अभाज्य संख्या वाले एक तरफ
- 2 व 3 से भाजित होने वाली संख्या वाले एक तरफ व अन्य एक तरफ

यह गतिविधि हो जाने के बाद दो शिक्षकों से पूछा कि वे कितने शिक्षकों के नाम जान पाये तो अधिकांश शिक्षकों से वे परिचित हो गये थे। गतिविधि के दौरान प्रतिभागियों के बीच 0 एवं 1 को लेकर चर्चा हुई जिसमे मुद्दा था कि क्या शून्य व एक सम है या विषम, भाज्य है या अभाज्य इसपर सुगमकर्ता द्वारा सम-विषम व भाज्य-

अभाज्य की समझ पर बातचीत करी गयी जिसमे मुख्य बातें निम्न रहीं:

सम-विषम, भाज्य-अभाज्य जैसे वर्गीकरण किस संख्या डोमेन में पढ़े जाते हैं?

उत्तर- प्राकृतिक संख्याओं में इसलिए शून्य को न तो हम सम-विषम और ना ही भाज्य-अभाज्य के वर्गीकरण में देखते हैं

सम-विषम संख्याओं को कैसे परिभाषित करते हैं?

उत्तर. सम संख्याएं मतलब वे संख्याएं जो 2 से भाजित हो जाती हैं तथा इनके अलावा बची संख्याएं विषम होती हैं अतः सबसे छोटी सम संख्या 2 हुई व 1 न तो सम और न ही विषम संख्या होता है

- भाज्य-अभाज्य संख्याओं को कैसे परिभाषित करते हैं?

उत्तर. अभाज्य संख्याएं मतलब वे संख्याएं जो केवल दो संख्याओं से भाजित होती हैं जिनमे से एक वह स्वयं होती है व दूसरी संख्या 1 होती है।अन्य संख्याएं जिनके दो से अधिक विशिष्ट भाजक हों वे भाज्य संख्याएं होती हैं।

इस गतिविधि के बाद बड़े समूह में एक वर्कशीट दी गई और उसको हल करने को कहा गया।

शिक्षकों द्वारा शीट को भर लेने के बाद अपने बगल में बेठे साथी को देते हुए शीट को चैक करने को कहा गया और उस पर नंबर देने को कहा गया।

तीसरी गतिविधि के रूप में सभी को अपनी कापी में 5×5 का एक ग्रिड बनाने को कहा जिसमें एक से सौ तक नंबर थे और सुगमकर्ता ने मैथ्सी नाम से इस खेल को खेला। हॉउजी गेम 5×5 खानों से संबंधित गणित के इस खेल में दिए गए 25 खण्डों में 1 से 99 के बीच का कोई भी अंक अपने मन से लिखने का निर्देश प्रत्येक प्रतिभागी को दिया गया। यह निर्देश दिये गये कि प्रत्येक प्रतिभागी यह ध्यान रखे कि किसी भी संख्या की पुनरावृत्ति न हो। अब दी गई संख्याओं को एक-एक कर प्राप्त निर्देशानुसार काटते हैं। किसी भी प्रतिभागी के एक लाईन आड़ी या खड़ी पूर्ण रूप से कट जाए उसे श्यामपट्ट पर लिखेगें तथा अन्य प्रतिभागियों से मिलान करवाया जायेगा। लाईन पूर्ण हो जाने पर उसे प्रोत्साहित करते हुए एक टाफी दी गई और खेल को इसी प्रकार आगे बढ़ाया गया।

हॉउजी हेतु कुछ निर्देश निम्न रहे:-

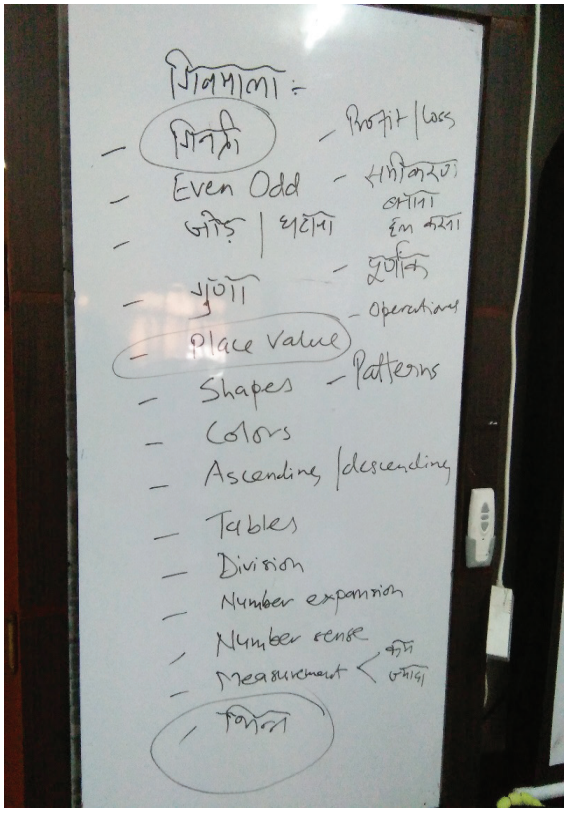
- इकाई में 5 और दहाई में 7 हो तो काट दीजिए।
- इकाई में 3 दहाई में दुगुना हो तो काट दीजिए।
- सबसे छोटी अभाज्य संख्या काट दें।
- 1 से 20 के बीच सभी पूर्ण वर्ग संख्याएं काट दें।
- वह संख्या जिसकी इकाई व दहाई का योग 7 हो काट दें।
- 1 से 99 तक की सबसे बड़ी अभाज्य संख्या काट दें।
- 13 के सभी गुणज काट दें।
- इकाई व दहाई के अंक का जोड़ 9 हो तो काट दें।
- 55 - 26 संक्रिया का हल काट दें।
- 25 + 58 संक्रिया का हल काट दें।
- 8 × 4 संक्रिया का हल काट दें।

गिनमाला के रूप में चैथी गतिविधि कराई गई जिसका मुख्य उददेश्य था कि गिनमाला के माध्यम से गणित के बिन्दुओं पर कैसे समझ बनाई जा सकती है

हांलाकि यह गतिविधि कुछ शिक्षक अपने विद्यालयों में अपना रहे हैं लेकिन यहां पर इस प्रक्रिया को कराने से शिक्षकों ने अपनी शंकाओं को स्पष्ट किया।

अगली गतिविधि के रूप में एक शिक्षक ने चाक के माध्यम से भिन्न को समझाने का प्रयास किया कि TLM के रूप में हमारे पास सबसे आसान माध्यम चॉक होता है जिस पर हमारी आसानी से पहुँच होती है इसके माध्यम से भिन्न को कैसे कक्षा कक्ष तक ले जा सकते हैं। जिस पर सुगमकर्ता ने तर्क दिया अगर हम चाक से 1/2 को दिखाते

हैं तो बीच से आधा करने पर क्या वाकई में वो आधा हो जाता है? किस रुप में आधा होता है? जिस पर बातचीत को आगे बढ़ाते हुए बात की गई कि हम लंबाई का आधा कर रहे हैं या आयतन का?



पांचवी गतिविधि के रूप में एक विडियो क्लिप दिखाया गया जिसमें पूर्णांक को सरल तरीके से पढ़ाने की विधि थी। सुगमकर्ता द्वारा बताया गया कि पूर्णांक का प्रयोग जोड़, घटाने की प्रक्रिया में कैसे होता है और इसे सरलता से कैसे प्रयोग किया जा सकता है।

जिसके उपरांत सुगमकर्ता ने प्राइमरी स्तर पर गणित को पढ़ाने के लिए टी.एल.एम. की उपयोगिता पर बल दिया जो कि उपरोक्त गतिविधियाँ करने से समझ में आता है।

अंत में सुगमकर्ता द्वारा निम्न बिन्दुओं को ध्यान में रखते हुए समेकन किया गया-

- TLM का प्रयोग कब, कैसे, किस तरह का होना चाहिए यह शिक्षक की अपनी समझ पर आधारित होता है। हमें यह कोशिश करनी चाहिए कि जो भी TLM का प्रयोग कर रहे हैं वह उस अवधारणा को समझाने में मदद कर रहा है या नहीं इस पर समझ होना बहुत जरूरी होना चाहिए।
- TLM केवल प्रिंट समाग्री ही नहीं बल्कि उसके लिए हम किसी विडियो, बच्चों आदि का उपयोग भी कर सकते हैं।
- प्राथमिक स्तर पर इसकी बहुत उपयोगिता है क्योंकि गणित एक अर्मूत विषय है जिस कारण से बिना संदर्भ व TLM के लिए बच्चों को इसे समझने में काफी दिक्कतों का सामना करना पड़ता है।
- किसी अवधारणा के लिए केवल एक तरह का TLM का उपयोग करना ही पर्याप्त नहीं इसलिए प्राथमिक स्तर पर बच्चों को अलग-अलग तरह के अनुभव दिये जाने चाहिए ताकि उनकी अवधारणा की समझ पुख्ता हो सके।
- गणित केवल अर्मूत विषय नहीं है बल्कि उसकी खुद की भाषा है, तार्किकता है व उसमें सोपानक्रम भी है इसलिए यह आवश्यक है कि इन सारे बिन्दुओं पर भी ध्यान दिया जाना चाहिए।
- इस समय प्राथमिक स्तर पर आ0 भा0 चि0 प्र0 यानि अनुभव, भाषा, चित्र व प्रतीक को सीखने सिखाने की पद्धति के तौर पर इस्तेमाल किया जा रहा है।
- किसी भी गतिविधि को चुनते वक्त निम्न बातों पर ध्यान दिया जाना चाहिए-
- गतिविधियों का चुनाव करते वक्त यह ध्यान रखना आवश्यक है कि गतिविधि अवधारणा विकसित करने के लिए है, अभ्यास के लिए है या फिर मूल्यांकन के लिए।
- गतिविधियाँ विषय की पाठ्यचर्या के अनरुप हों।
- गतिविधियों को बच्चों के स्तर के अनुसार, विषय की प्रकृति के अनुसार, अवधारणा के अनुरुप चुना जाना चाहिए।
- खासतौर पर खुद से निर्माण की गयी सामग्री की बहुत उपयोगिता है क्योंकि शिक्षक व बच्चे उससे सामाजिक व अपनत्व के भाव से जुड़ते हैं।

♦ गतिविधियों के लिए निर्देश स्पष्ट होना चाहिए।

उपरोक्त गतिविधियाँ हो जाने के बाद शिक्षकों की ओर से -

इतने कम समय में हमें जो भी गतिविधियाँ कराई गई वे काफी सरल और विद्यालय में कर पाने योग्य थी। क्योंकि कई बार हम सोचते हैं कि TLM के लिए सामान नहीं है कैसे बनाया जाए। इन गतिविधियों से हमें लग रहा है कि कानसेप्ट की समझ यदि है तो सामग्री का प्रयोग तो कैसे भी किया जा सकता है।

पूर्णांक वाली गतिविधि प्राइमरी स्तर पर नहीं हैं लेकिन यदि शिक्षक गणित पढ़ा रहा है तो इस पर शिक्षक की समझ होनी चाहिए।

गणित को लेकर हमारे मन में कई सवाल भी थे यहाँ पर माहौल मिला तो हमने बोल दिया हाँलाकि अभी कई बिन्दु हैं जिन पर हम आगे बात करेगें। हमारी ओर से कोशिश रही कि जो अपेक्षाएं शुरूआत में उनकी ओर से आईं उसे आगे साथ मिलकर हल करने की ओर जाने का वादा किया। इसके साथ ही सत्र का समापन हो गया।

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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