D 103769 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2024

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Not more than 25 marks can be earned from this Section.
Each question carries 2 marks.

1. What is the natural domain of the function $f(x) = x^2$. Is the function one-to-one ? Justify your answer.

2. Determine whether the function $f(x) = x \sin x$, even, odd or neither even nor odd.

3. Find $(f \circ g \circ h)(x)$ if $f(x) = \sqrt{x}$, $g(x) = 1/x$, $h(x) = x^3$.

4. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2 + 100} - 10}{x^2}$.

5. The area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter is 10 m ?

6. Show that when x is very near 0, and k is any real number, then

   $(1 + x)^k \approx 1 + kx$.

7. Find dy and Δy at $x = 3$ with $dx = \Delta x = 2$ where $y = \sqrt{x}$.

8. State Rolle's Theorem.

Turn over

9. Is $x^5 - x^3 - 2x^2$ increasing or decreasing at -2 ? Justify.

10. For what values of x is the curve $y = 2\sqrt{ax}$ concave to the foot of the ordinate.

11. Find $\int (x+2)\left(x^2-1\right) dx$.

12. Show that $\int_a^b x dx = \frac{b^2 - a^2}{2}$.

13. Show that if f is continuous on $[a, b], a \neq b,$ and if $\int_a^b f(x)\, dx = 0,$ then $f(x) = 0$ at least once in $[a, b]$.

14. State the Fundamental Theorem of Calculus part-1

15. Find the work done in lifting a 1000 lb object 1.25 ft off the ground.

Section B

Not more than 35 marks can be earned from this Section.
Each question carries 5 marks.

16. State The Squeeze Theorem. Use the same to evaluate $\lim\limits_{x \to 0} x^2 \sin \frac{1}{x}$.

17. Find the local linear approximation of $f(x) = \sqrt{x}$ at $x = x_0 = 9$ and use it to approximate $\sqrt{9.02}$, $\sqrt{8.82}$ and $\sqrt{10}$. Also find absolute error

18. Prove that if $f'(x) = 0$ for all x in an interval (a, b) then f is constant on (a, b).

19. Find $\lim\limits_{x \to +\infty} \frac{\sqrt{x^2+3}}{5x-6}$.

20. In a test run of a high-speed train along a straight elevated monorail track, data obtained from reading its speedometer indicated that the velocity (in ft/sec) of the train at time t can be described by the velocity function

$v(t) = 7.8\,t \quad 0 \le t \le 25.$

Find the position function of the train. Assume that the maglev is initially located at the origin of a co-ordinate line.

21. Find $\dfrac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt.$

22. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.

23. Find the center of mass of a system comprising four particles with masses 6, 2, 3, and 5 slugs, located at the points $(-1, 3), (-2, -1), (2, 6)$ and $(5, 1)$, respectively. (Assume that all distances are measured.

Section C

Answer any **two** *question.*
Each question carries 10 *marks.*

24. (a) State and prove the Lagrange's Mean Value Theorem

(b) Verify that the following functions satisfies the hypothesis of mean value theorem on the given internal and find all value of c $f(x) = x^2, [0, 2]$.

25. Sketch a graph of

$f(x) = x^3 - 3x^2 + 1.$

26. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence ?

27. (a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(b) For the curve $y = c \cosh \dfrac{x}{c}$, , show that $y^2 = c^2 + s^2$, where s is the length of the arc measured from its vertex to the point (x, y).

(2 × 10 = 20 marks)