D 103770 (Pages : 3) Name....................................

Reg. No................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions Only)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Maximum marks : 20.

1. Describe the set of points P whose polar coordinates (r, θ) satisfy $0 \le r \le 2$ and $0 \le \theta < r$.

2. Differentiate : (a) $\left(\tan 3x\right)\big/1 + \sin^2 x$; (b) $1 - \csc^2 5x$.

3. Show that $e^x = \cosh x + \sinh x$.

4. Find $\int_1^b \frac{1}{x^4}\, dx$. What happens as b goes to infinity ?

5. State the comparison test for integrals.

6. Briefly explain Taylor's and Maclaurin's series.

7. Write down the criteria for checking whether a given subset W is a subspace of a vector space V.

8. Define linear independence.

9. State a condition for the consistency of the matrix equation AX = B.

10. Find the inverse of $A = \begin{pmatrix} 1 & 4 \\ 2 & 10 \end{pmatrix}$.

11. Define the adjoint of an $n \times n$ matrix.

12. State Cayley Hamilton theorem.

Turn over

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum marks : 30.

13. Convert from cartesian to polar co-ordinates : $(2, -4)$; and from polar to cartesian coordinates : $(6, -\pi/8)$.

14. Calculate : (a) $\dfrac{d}{dx}\sinh^{-1}(3x)$; and (b) $\dfrac{d}{dx}[\sinh^{-1}(3\tanh 3x)]$.

15. (a) For which values of the exponent r is $\int_1^\infty x^r\, dx$ convergent ?

    (b) Find $\int_{-\infty}^\infty \dfrac{dx}{1+x^2}$.

16. Let $f(x) = \cos x$. Evaluate $\int_0^{\pi/2} \cos x\, dx$. by the method of Riemann sums, taking 10 equally spaced points : $x_0 = 0,\ x_1 = \pi/20,\ x_2 = 2\pi/20,\ ...,\ x_{10} = 10\pi/20 = \pi/2$ and $c_i = x_i$. Compare the answer with the actual value.

17. Define a vector space.

18. Reduce to echelon form the augmented matrix :

$$\begin{pmatrix} 2 & 6 & 1 & | & 7 \\ 1 & 2 & -1 & | & -1 \\ 5 & 7 & -4 & | & 9 \end{pmatrix}.$$

19. Find the inverse of $A = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum 10 *marks.*

20. Describe Newton's Method for solving $f(x) = 0$.

Use Newton's method to find the first few approximations to a solution of the equation $x^2 = 4$, taking $x_0 = 1$.

21. (a) Describe Gram Schmidt Process in R^2 and R^3.

(b) Orthonormalize $\mathrm{B} = \{u_1, u_2\}$, where $u_1 = \langle 3, 1 \rangle$, $u_2 = \langle 1, 1 \rangle$.

(c) Orthonormalize $u_1 = \langle 1, 1, 1 \rangle$, $u_2 = \langle 1, 2, 2 \rangle$, $u_3 = \langle 1, 1, 0 \rangle$.