D 103062 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Type Question)

All questions can be attended.
Each question carries 2 marks.
Overall ceiling 25.

1. Give an example of a system of linear equation with the following properties :
   (i) Unique solution
   (ii) Infinite number of solutions
2. Solve the system $x + y = 2, x - y = 0$ by using any method.
3. Give an example to show that the matrix multiplication is need not be commutative.
4. Find the row reduced echelon form of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}.$$

5. Let $W = \{(x, y, z) : x + y + z = 0\}$. Show that W is subspace of $\mathbb{R}^3$.
6. Show that $\{(1,0),(0,1)\}$ spans $\mathbb{R}^2$.
7. Define Wronskian. Find the Wronskian of $\sin 5x$ and $\cos 5x$.
8. Define linearly independent set. Give an example.
9. Define row space and column space of a matrix.

Turn over

10. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

11. Show that the operator $T : \mathbb{R}^2 \to \mathbb{R}^2$, that projects onto the x-axis in the xy-plane is not one-one.

12. Find the eigen values of $\begin{bmatrix} 3 & 0 \\ 8 & 1 \end{bmatrix}$.

13. Define similar matrices. Show that if A and B are similar the determinant is equal.

14. Let $\boldsymbol{u} = U = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \boldsymbol{v} = V = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$. Evaluate $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$, where, $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \langle U, V \rangle = trace\left(U^T V\right)$.

15. Define orthogonal matrix. Show that $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is orthogonal.

(Ceiling 25 Marks)

Section B (Paragraph/Problem Type Questions)

All questions can be attended.
Each question carries 5 marks.
Overall Ceiling 35.

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, find, det (A), A^{-1}, A^{-2}, A^{-3} and A^{-5}.

17. Using row reduction, evaluate the determinant of :

$$\begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}.$$

18. Determine the set $\{6, 3\sin^2 x, 2\cos^2 x\}$ is independent or not.

19. Show that the matrices

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Form a basis for the vector space M_{22} of 2×2 matrices.

20. Find a basis for row space of the matrix

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}.$$

21. Describe the null space of the matrix $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$.

22. Define inner product. Consider P_2 with the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$. Verify that x and x^2 are orthogonal with respective above inner product.

23. Let $f = f(x)$ and $g = g(x)$ be two functions on $C[a, b]$. Show that $\langle f, g \rangle = \int_a^b f(x)\, f(x)\, dx$ defines an inner product on $C[a, b]$.

(Ceiling 35 Marks)

Section C (Essay Type Question)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) For what values of b_1, b_2 and b_3 the following system of equations are consistent ?

$$x_1 + x_2 + 2x_3 = b_1$$
$$x_1 + 0x_2 + x_3 = b_2$$
$$2x_1 + x_2 + 3x_3 = b_3.$$

(b) Let A and B are symmetric matrices of same size. Then show that the followings.

(i) A^T is symmetric

(ii) A + B and A – B are symmetric

(iii) kA is symmetric, where k is any scalar.

25. Let $u = \{1, 2, -1\}$, $v = \{6, 4, 2\}$ in $\mathbb{R}^3$.

(a) Show that $w = \{9, 2, 7\}$ is in the linear combination of u and v.

(b) Show that $w = \{4, -1, 8\}$ is not in the linear combination of u and v.

26. Consider the matrix,

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

(a) Verify that $\operatorname{rank}(A) = \operatorname{rank}\left(A^T\right)$.

(b) Verify dimension theorem for the matrix A.

27. Find an orthogonal matrix P that diagonalizes :

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}.$$

(2 × 10 = 20 marks)