D 100614 **(Pages : 3)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions onwards)

Time : Two Hours and a Half — Maximum Marks : 80

Section A

Questions 1—15. *Answer any number of questions. Each carry* 2 *marks. Maximum marks* 25.

1. Find the domain of the function $f(x,y)=\dfrac{\ln(x+y+1)}{y-x}$.

2. Evaluate $\operatorname*{Lt}\limits_{(x,y)\to(1,2)} \dfrac{2x^2-3y^3+4}{3-xy}$.

3. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $\ln\left(x^2+y^2\right)+yz^3+2x^2=10$.

4. Find the gradient of $f(x,y)=x^2+y^2+1$ at the point (1, 2). Use the resutl to find the directional derivative of f at (1, 2) in the direction from (1, 2) to (2, 3).

5. Find the equation of the tangent plane to the hyperboloid $z^2-2x^2-2y^2=12$ at the point $(1,-1,4)$.

6. Find the critical points of $f(x,y)=-x^3+4xy-2y^2+1$.

7. Evaluate $\int_0^2\int_{y^2}^4 dx\,dy$.

8. Set up a triple integral for the volume of the solid region in the first octant bounded above by the sphere $x^2+y^2+z^2=6$ and below by the parabolid $z=x^2+y^2$.

9. Evaluate $\iint\limits_R 1-2xy^2 dA$ where R is the regoin $\{(x,y)\,|\,0\le x\le 2,-1\le y\le 1\}$.

Turn over

10. Find the volume of the solid S below the hemisphere $z = \sqrt{9 - x^2 - y^2}$ above the xy plane and inside the cylinder $x^2 + y^2 = 1$.

11. Find the gradient vector field $\vec{F}$ of the function $f(x, y, z) = \frac{-K}{\sqrt{x^2 + y^2 + z^2}}$ and hence deduce that the inverse square field $\vec{F}$ is conservative.

12. Find a parametric representation for the cone $x^2 + y^2 = z^2$.

13. State Stoke's theorem.

14. Using Divergence theorem evaluate $\iint_S \vec{F}.n \, dS$ where $\vec{F} = x\hat{i} + y^2\hat{j} + z\hat{k}$ and S is the surface bounded by the co-ordinate planes and the plane $2x + 2y + z = 6$.

15. Find an equation of the tangent plane to the paraboloid $\vec{r}(u, v) = u\hat{i} + v\hat{j} + (u^2 + v^2)\hat{k}$ at the point (1, 2, 5).

Section B

Questions 16—23. *Answer any number of questions.*
Each carry 5 *marks. Maximum marks* 35.

16. Let $w = 2x^2y$ where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

17. The dimensions of a closed rectangular box are measured as 30 in 40 in and 60 in with a maximum error of 0.2 inches in each measurement. Using differentials find the maximum error in calculating the volume of the box.

18. Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at (x_0, y_0, z_0) is $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$.

19. Sketch the level curve corresponding to $c = 0$ for the function $f(x, y) = y - \sin x$ and find a normal vector at the point $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$.

20. Using polar co-ordinates find the volume of the solid region bounded above by the hemisphere $z = \sqrt{16 - x^2 - y^2}$ and below by the circular region $x^2 + y^2 \leq 4$.

21. Find the surface area S of the portion of the hemisphere $f(x, y) = \sqrt{25 - x^2 - y^2}$ that lies above the region R bounded by the circle $x^2 + y^2 \leq 9$.

22. Evaluate $\oint_C \left(y^2 + \tan x\right) dx + \left(x^3 + 2xy + \sqrt{y}\right) dy$ where C is the circle $x^2 + y^2 = 4$ and is oriented in the positive direction.

23. Find the surface area of the unit sphere $\vec{r}(u, v) = \sin u \cos v\hat{i} + \sin u \sin v\hat{j} + \cos u\hat{k}$ where the domain D is $0 \leq u \leq \pi$ and $0 \leq v \leq 2\pi$.

Section C

Questions 24—27. *Answer any* **two** *questions.*
Each carry 10 *marks.*

24. (a) Show that $\lim_{(x,y)\to(0,0)} \dfrac{2x^2 y}{x^2 + y^2} = 0.$

    (b) The production function of a certain company is $f(x, y) = 20x^{2/3}y^{1/3}$ Billion dollars, when x billion dollars of labour and y billion dollars of capital are spent :

    (i) Compute $f_x(x, y), f_y(x, y)$.

    (ii) Compute f_x (125, 27) and f_y (125, 27) and interpret your result.

25. Let $T(x, y, z) = 20 + 2x + 2y + z^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 11$. Find the extreme temperatures on the curve formed by the intersection of the plane $x + y + z = 3$ and the sphere.

26. Find the volume of the solid that lies below the paraboloid $z = 4 - x^2 - y^2$ above the xy plane and inside the cylinder $(x - 1)^2 + y^2 = 1$.

27. Verify Stoke's theorem for the vector field $\vec{F}(x, y, z) = 2z\hat{i} + 3x\hat{j} + 5y\hat{k}$ and S is the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \leq 0$ with upward orientation and C is the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of S in the xy-plane.

(2 × 10 = 20 marks)