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D 100613 **(Pages : 2)** **Name.................................**

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admissions onwards)

Time : Two Hours and a Half　　　　Maximum Marks : 80

Section A

Answer any number of questions.
Each question carries 2 marks. Maximum marks 25.

1. Show that $f(z)=\overline{z}$ is nowhere differentiable.
2. Verify Cauchy-Riemann equations for $f(z) = z^2$.
3. Write the Cauchy-Riemann equations in polar co-ordinates.
4. Define harmonic function and harmonic conjugate function.
5. Solve $e^w = -2$.
6. Express $\cos(2-4i)$ in the form $a + ib$.
7. Evaluate $\int y\,dx + x\,dy$ on the curve $y = x^2$ from (0, 0) to (1, 1).
8. Evaluate $\oint z\,dz$ over the first quadrant of the circle $|z| = 1$ from $z = \mathrm{i}$ *to* $z = 1$.
9. Prove that $\int_C f(z)dz = 0$ for $f(z)=\dfrac{z^2}{z-3}$ where C is the unit circle $|z| = 1$.
10. Evaluate $\oint_C \dfrac{z+1}{z^4+2iz^3}\,dz$ where C is the circle $|z| = 1$.
11. Define (a) Power series ; (b) Circle of convergence.
12. Write the Maclaurin series expansion for $\sin z$ and $\cos z$.
13. Expand $f(z)=e^{\frac{3}{z}}$ in a Laurent series valid for $0<|z|<\infty$.
14. Find zeroes of $f(z)=\dfrac{z-2}{z^2}\sin\dfrac{1}{z-1}$.
15. Find the residue of $f(z)=\dfrac{z}{z^2+1}$ at its poles.

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Section B

Answer any number of questions.
Each question carries 5 marks. Maximum marks 35.

16. Verify the Cauchy Riemann equations for $f(x+iy) = \dfrac{x-iy}{x^2+y^2}$.

17. If $f(z) = u + iv$ then show that $|f'(z)|^2 = u_x^2 + v_x^2 = u_y^2 + u_y^2$.

18. Find the image of the annulus $2 \leq |z| \leq 4$ under the mapping w = Ln z.

19. Find the principal value of $(-3)^{\frac{i}{\pi}}$.

20. Using Cauchy-Goursat theorem, evaluate $\oint_C \dfrac{5z+7}{z^2+2z-3}dz$ where C is the circle $|z-2| = 2$.

21. Find $\oint_C \dfrac{z^2-4z+4}{z+i}dz$ where C is the circle $|z| = 2$.

22. Prove that the sequence $\left\{\dfrac{3+ni}{n+2ni}\right\}$ converges to $\dfrac{2}{5}+\dfrac{1}{5}i$.

23. Find the residue of $f(z) = \tan z$ at $z = \dfrac{\pi}{2}$.

Section C

Answer any **two** *questions.*
Each question carries 10 marks. Maximum marks 20.

24. Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane and find the harmonic conjugate function.

25. State and prove (a) Liouville's theorem ; (b) Morerea's thereom.

26. State Cauchy's residue theorem, and using this show that $\int_C \dfrac{dz}{z\sin z} = 0$, where C is the unit circle about the origin described in the positive sense.

27. Show that $\int_0^{2\pi} \dfrac{d\theta}{2+\cos\theta} \dfrac{2\pi}{\sqrt{3}}$.

(2 × 10 = 20 marks)