D 100617 **(Pages : 2)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2019 Admission onwards)

Time : Two Hours Maximum Marks : 60

Section A (Short Answer Type Questions)

Answer any number of questions.
Each question carries 2 marks. Maximum marks 20.

1. Find the number of edges of $k_{2,3}$.
2. Draw the graph $K_5-\{e\}$.
3. Define degree of a vertex. Explain with example.
4. Let G be a simple graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ? Justify.
5. Give an example of a self-complementary graph with five vertices.
6. Let G be a simple graph with n vertices and $\overline{G}$ be its complement. Prove that, for each vertex V in G, $d_G(v)+d_{\overline{G}}(v)=n-1$.
7. A connected graph G has 21 vertices, what is the minimum possible number of edges in G.
8. Define diameter of a graph G. Which simple graphs have diameter 1 ?
9. When can you say that the wheel graph $W_n, n \geq 4$ is Euler ? Justify.
10. Define Jordan curve. Give an example.
11. Define Spanning tree. State Cayleys theorem in spanning trees.
12. Let G be a Hamiltonian graph. Show that G does not have a cut vertex.

Section B (Paragraph/Problem Type Questions)

Answer any number of questions.
Each question carries 5 marks. Maximum marks 30.

13. Prove that k_5, the complete graph on five vertices, is non-planar.
14. Let G be a planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.

Turn over

15. Explain Konigsberg bridge problem.
16. Let G be a graph in which the degree of every vertex is at least two then prove that G contains a cycle.
17. Prove that a vertex V of a tree T is a cut vertex if and only if $d(v) > 1$.
18. Let G be a connected graph, then G is a tree if and only if every edge of G is a bridge.
19. Given any two vertices u and v of a graph G, prove that every u-v walk contains u-v path.

Section C (Essay Type Questions)

Answer any **one** *questions.*
The question carries 10 *marks.*

20. Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.
21. Prove that if T is a tree with n vertices then it has precisely n-1 edges.

(1 × 10 = 10 marks)