D 53677 **(Pages : 2)** **Name..................................**

Reg. No..............................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Does the statement "$x + 2 = 5$" is a proposition ? Justify your answer.
2. Define the logical operator "disjunction" and illustrate with an example.
3. Evaluate the Boolean expression $\sim[a > b] \wedge (b < c)$ for $a = 3$, $b = 5$ and $c = 6$.
4. Prove that there is no positive integer between 0 and 1.
5. Prove that $1 + 2 +\text{------}+ n = \dfrac{n(n+1)}{2}$.
6. Show that $n^3 - 3n^2 + 2n$ is divisible by 2.
7. Prove that a product of 3 consecutive natural numbers is divisible by 6.
8. Determine whether 1661 is a prime or not.
9. Prove or disprove : $n! + 1$ is a prime for every positive integer n.
10. Let $(a, b) = d$. Prove that $(a/d, b/d) = 1$
11. Express (12, 28) as a linear combination of 12 and 28..
12. Prove the following : if p is a prime and p/ab. Then p/a or p/b.
13. Find the 1 cm. of 1050 and 574 .

Turn over

14. Is $99 \equiv 7\ (m0d3)$?. Justify your answer.

15. Show that $2^{32} + 1$ is divisible by 641.

Section B

Answer any number of questions.
Each question carries 5 marks.
Overall Ceiling is 35.

16. Find the reminder when $(n^2 + n + 41)^2$ is divided by 12.

17. Solve $12x \equiv 48(\text{mod}18)$.

18. Let a, b are positive integers. Prove that $[a, b]\ (a, b) = ab$.

19. Find the general solution of the LDE $6x + 8y + 12z = 10$.

20. Prove that there are infinitely many primes.

21. State and Prove the Pigeonhole Principle.

22. Construct a truth table for $(p \to q) \leftrightarrow (\sim p \vee q)$.

23. State and prove the associate laws for Conjunction and Dis junction.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Prove that there is a positive integer that can be expressed in two different ways as the sum of two cubes.

    (b) Prove that there is a prime number > 3.

    (c) Prove by contradiction, $\sqrt{2}$ is irrational.

25. State and Prove the Division Algorithm.

26. State and Prove The Fundamental theorem of Arithmetic.

27. (a) Find the reminder when $1! + 2! + ... + 100!$ is divided by 15.

    (b) Show that $11.14^n + 1$ is a composite number.

(2 × 10 = 20 marks)