D 103063 (Pages : 3) Name..................................

Reg. No...............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2024

Mathematics

MTS4C04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve $dx + e^{3x}\, dy = 0$.

2. Find general solution of $x\dfrac{dy}{dx} - 4y = x^6 e^x$.

3. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y\left(1 - x^2\right)}$, $y(0) = 2$.

4. Determine whether the functions $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = 4x - 3x^2$ are linearly dependent or linearly independent on the interval $(-\infty, \infty)$.

5. The function $y_1 = \ln x$ is a solution of $xy'' + y' = 0$. Find a second solution $y_2(x)$.

6. Solve the initial value problem $4y'' + 4y' + 17y = 0$, $y(0) = -1$, $y'(0) = 2$.

7. Find a homogeneous linear differential equation with constant co-efficient whose solution is $y = c_1 \cos 8x + c_2 \sin 8x$.

8. Let $f(t) = \begin{cases} t, & 0 \le t \le 1 \\ 1, & t \ge 1 \end{cases}$. Find $\mathcal{L}(f(t))$.

Turn over

9. Find $\mathscr{L}^{-1}\left(\frac{4s}{4s^2+1}\right)$.

10. Evaluate $\mathscr{L}(t \sin kt)$.

11. Show that the functions $f_1(x) = x$, $f_2(x) = \cos 2x$ are orthogonal on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - 6\frac{\partial u}{\partial y} = 0$ is parabolic .

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 30.

13. Solve $\left(x^2 + y^2\right) dx + \left(x^2 - xy\right) dy = 0$.

14. Solve the initial value problem $\frac{dy}{dx} = \cos(x + y)$, $y(0) = \frac{\pi}{4}$.

15. Solve $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$.

16. Solve the homogeneous boundary value problem $y'' + \lambda y = 0$, $y(0)$, $y(L) = 0$.

17. Solve $x^2y'' - 3xy' + 3y = 2x^4e^x$.

18. Solve $f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau)\, e^{t-\tau}\, d\tau$ for $f(t)$.

19. Expand $f(x) = x^2$, $0 < x < L$ in a cosine series.

Section C

Answer any **one** *questions.*

Each question carries 10 *marks.*

20. Solve the initial value problem $y'' + 4y' + 6y = 1 + e^{-t}$, $y(0) = 0$, $y'(0) = 0$ using Laplace transform.

21. Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2, & 0 \le x < \pi \end{cases}$ on the interval $[-\pi, \pi]$.